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APPROXIMATION OF KOBE POSTERIORS FROM STOCK SYNTHESIS FOR NORTH ATLANTIC BLUE SHARK

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SUMMARY

The SCRS provides probabilistic statements about relative stock status (SSB/SSB_{MSY}) and exploitation (F/F_{MSY}), using the Kobe phase plot and strategy matrix. For integrated models, probability distributions are generally constructed using bootstrap or Markov Chain Monte-Carlo (MCMC) methods, which are computationally intensive and make it challenging to run during the short time frame of assessment meetings. As a case in point, there was insufficient time during the Blue Shark Stock Assessment Meeting to complete the probabilistic advice, especially as the code had not been verified nor the procedure formally validated. This paper, therefore, demonstrates how to provide probabilistic estimates in real-time for the current and future status of North Atlantic blue shark. This is done using the delta-multivariate log-normal approximation (MVLN) to derive posteriors from Stock Synthesis outputs. We document the method and verify the code by comparing the derived estimates with the covariance matrix from Stock Synthesis. Furthermore, we compare the MVLN estimates to those obtained using MCMC. However, our results are preliminary and are provided here only as example implementations solely to demonstrate possible implementations of the methods.

RÉSUMÉ

Le SCRS formule des avis probabilistes sur l'état relatif (SSB/SSB_{PME}) et l'exploitation (F/F_{MSY}) du stock à l'aide du diagramme de phase de Kobe et de sa matrice de stratégie. Pour les modèles intégrés, les distributions de probabilité sont généralement élaborées en utilisant des méthodes par bootstrap ou de Markov Chain Monte-Carlo (MCMC) à forte intensité de calcul qui sont difficiles à exécuter pendant le court laps de temps des réunions d'évaluation. Par exemple, la réunion d'évaluation du stock de requin peau bleue n'avait pas disposé du temps suffisant pour achever l'avis probabiliste, notamment étant donné que le code n'avait pas été vérifié et que la procédure n'avait pas été officiellement validée. Ce document démontre, par conséquent, comment fournir des estimations probabilistes en temps réel pour l'état actuel et futur du requin peau bleue de l'Atlantique Nord. Cela est réalisé en utilisant l'approximation lognormale delta-multivariée (MVLN) pour déduire les distributions a posteriori des données de sortie de Stock Synthesis. Nous documentons la méthode et vérifions le code en comparant les estimations déduites avec la matrice de covariance de Stock Synthesis. En outre, nous comparons les estimations de MVLN avec celles obtenues en utilisant MCMC. Toutefois, nos résultats sont préliminaires et ne sont ici fournis que comme un exemple d'applications dans le seul but de démontrer des applications possibles de ces méthodes.

RESUMEN

El SCRS realiza afirmaciones probabilísticas sobre el estado relativo del stock (SSB/SSB_{RMS}) y la explotación (F/F_{RMS}), utilizando el diagrama de fases de Kobe y la matriz de estrategia. En el caso de los modelos integrados, las distribuciones de probabilidad se construyen generalmente utilizando los métodos bootstrap o Markov Chain Monte-Carlo (MCMC), que son intensivos desde el punto de vista informático y dificultan su ejecución en el breve plazo de tiempo de las reuniones de evaluación. Por ejemplo, en la reunión de evaluación del stock de tiburón azul no hubo tiempo suficiente para completar el asesoramiento probabilístico, especialmente, porque el código no se había verificado ni el procedimiento validado formalmente. Este documento demuestra, por tanto, cómo proporcionar estimaciones probabilísticas en tiempo real del estado actual y futuro del tiburón azul del Atlántico norte. Para ello se utiliza la aproximación multivariante log-normal (MVLN) para derivar los resultados de las distribuciones posteriores de Stock

Synthesis. Documentamos el método y verificamos el código comparando las estimaciones derivadas con la matriz de covarianza de Stock Synthesis. Además, comparamos las estimaciones MVLN con las obtenidas utilizando MCMC. No obstante, nuestros resultados son preliminares y se ofrecen exclusivamente como ejemplos de aplicación con el único fin de demostrar posibles implementaciones de los métodos.

KEYWORDS

Kobe phase plot, Kobe Strategy Matrix, Stock status, Uncertainty, Multivariate-lognormal (MVLN), Delta-MVLN method

1. Introduction

There was insufficient time at the 2023 blue shark stock assessment meeting (Anon 2023a) to fully develop, then verify the code and validate the probabilistic advice with the integrated assessment model, Stock Synthesis (SS3, Methot and, Wetzel, 2013). This paper, therefore, demonstrates how to provide probabilistic estimates about the stock status in real-time of the current and future status of North Atlantic blue shark. This is done using the delta-multivariate log-normal approximation (MVLN) to derive posteriors from SS3 outputs. We document the method and verify the code by comparing the derived estimates with the covariance matrix from SS3, which is widely applied across ICCAT and other tuna Regional Management Organizations. Furthermore, we compare the MVLN estimates to those obtained using MCMC. However, our results are preliminary and are provided only as an example implementation solely for the purpose of demonstrating possible implementations of the methods in the future.

Advice by the Standing Committee on Research and Statistics (SCRS) of ICCAT is provided using the Kobe phase plot to summarise the current status and the Kobe 2 Strategy Matrix (K2SM) to summarise the future state of the stock under alternative management options. Both the plot and the K2SM translate uncertainty about the stock relative to maximum sustainable yield (MSY) reference points, based on SSB/SSB_{MSY} and F/F_{MSY}, into probabilistic statements. The two main approaches applied to construct the joint probability distributions of SSB/SSB_{MSY} and F/F_{MSY} are i) running many models (i.e. a grid) with alternative parameterisation to capture structural, i.e. across-model uncertainty or ii) estimating the within-model uncertainty for a single base-case or reference model. Due to the inherently correlated nature of SSB/SSB_{MSY} and F/F_{MSY}, the probability distributions within a model are generally constructed using bootstrap or Markov Chain Monte-Carlo methods (MCMC).

Both the bootstrap and MCMC are computationally intensive and time-consuming to conduct for integrated assessments (Magnusson *et al*., 2013; Monnahan, *et al*., 2019). Potential problems include model misspecification, data conflicts, poor initialisation, and inadequate tuning, which renders them challenging tasks to complete within the short time frames of assessment meetings. This is especially true if they also have to be applied to grids of several models.

Models based on automatic differentiation can provide covariances of derived quantities at little computational cost. Therefore, an alternative is to use MVLN to account for covariance between F/F_{MSY} and SSB/SSB_{MSY}. A problem is that the covariance between these two quantities is approximated on the log scale, whereas SS3 outputs those on the natural scale. Walter *et al*. (2019), therefore provided an approximation for calculating Kobe posterior for $\log(SSB/SSB_{MSY})$ and $\log(F/F_{MSY})$ based on the maximum likelihood estimates (MLEs), standard errors (SEs) and the correlation of the untransformed quantities F/F_{MSY} and SSB/SSB_{MSY}.

For verification, MVLN and SS3 estimates can be compared, i.e. does the code replicate the covariances between derived quantities? These can also be compared to MCMC estimates, for both historical estimates and forecasts. Even if the code replicates the SS3 covariances, differences may be seen with the MCMC estimates. Potential problems are if the model is misspecified and parameters poorly estimated (Monnahan *et al*. 2019), or if there is not enough data for the number of parameters being estimated. Since the production function and reference points are largely determined by fixed parameters (e.g. natural mortality, steepness, Mangel *et al*., 2013), if priors are used for steepness, then the covariances estimated from the hessian may not be comparable to those from MCMC (Stewart *et al.* 2013).

The MLVN approach to approximate within-model uncertainty (Walter *et al.*, 2019) has only recently been introduced in ICCAT for use in the Kobe phase plot and K2SM. An advantage of the MVLN approach is the rapid generation of management advice during typically time-constrained tuna RFMO assessment meetings, and alternatives including the bootstrap and MCMC are much more computationally intensive. Consequently, the reduction in computing time of the MVLN approach allows rapid generation of management advice from a range of plausible alternative model configurations. For example, those developed from a structural uncertainty grid, show more variability than the typically narrower range of within-model uncertainty.

The MVLN approach described here is analogous to those previously described for other ICCAT SCRS species (Walter *et al*. 2019; Winker *et al.* 2019; Walter and Winker 2020). However, we demonstrate a specific application of the MVLN approach to the SCRS Shark Species Group 2023 blue shark assessment. In comparison, the MVLN methods described here have also previously been presented and discussed, although not adopted, within the SCRS Shark Species Group during the 2019 intersessional meeting held to update projections for shortfin mako based on the 2017 assessment (Anon. 2020, their SCRS/2019/093 [withdrawn] and their presentation SCRS/P/2019/035).

The MVLN approach (Walter *et al.*, 2019) has also been previously presented and discussed during the 2019 ICCAT Working Group on Stock Assessment Methods (WGSAM; Anon. 2019, their presentation SCRS/P/2019/020 and subsequent discussion). The WGSAM 2019 Group suggested that the MVLN method provides a promising solution that would allow producing the Kobe phase plot and K2SM in time for the adoption of the stock assessment report. The MVLN approach has subsequently been adopted within other SCRS working groups for the provision of management advice for several species (E.g., Anon. 2021, their Section 9.1 BET – Bigeye Tuna; Anon. 2023b, their Section 9.3 SKJ – Skipjack and Section 9.12 SWO-AT – Atlantic swordfish). However, the WGSAM 2019 Group recommended that more comprehensive comparisons between MVLN and MCMC and bootstrap approaches should be conducted before adopting the MVLN as the sole method of choice. Unfortunately, the resulting WGSAM 2019 recommendation and work plan items related to MVLN (see below) do not appear to have been implemented in subsequent WGSAM meetings, possibly as a result of COVID-19 or changes in personnel attending WGSAM.

WGSAM 2019 also made recommendations without financial implications (Anon. 2019, their Section 9):

> "3. The [WGSAM 2019] Group recommended that an analysis be conducted based on comparing past ICCAT (or other tRFMOs) assessments using Monte Carlo Markov Chain (MCMC) analysis or bootstrapping techniques to the multi-variate normal (MVN) methods presented during this meeting so that a determination can be made as to whether the MVN method is an effective and reliable option for producing equivalent results in a more efficient and timely manner."

WGSAM 2019 work plan items (Anon. 2019, their Section 9):

> "4. A comparison study of MCMC and bootstrapping to MVN techniques to characterize stock assessment uncertainty."

2. Material and Methods

We derive the variance and covariance of SSB/SSB_{MSY} and F/F_{MSY} based on the maximum likelihood estimates (MLEs). These quantities are functions of asymptotically normal random variables with known variance and are derived by SS3 using the delta method (Doob, 1935), we therefore refer to the approach as 'delta-Multivariate log-Normal' (delta-MVLN).

We then verify the delta-MVLN method by comparison to the estimates and the covariance generated by SS3. For simplicity, we focus on the use of the delta-MVLN method for within-model uncertainty, although it is also possible to extend the concept to multiple models. For assessment and projection setting, see the working group report (Anon., 2023a), which we attempted to replicate for the demonstrations provided here.

2.1 The delta-MVLN method

The Hessian is the matrix of second-order partial derivatives of the parameters with respect to the Maximum Likelihood. The delta method can be used to estimate the variance of a derived quantity and the covariance between quantities.

To generate Kobe posteriors from a MVLN distribution requires the means and the variance-covariance matrix (*VCM*) of $\log(SSB/SSB_{\mathrm{MSY}})$ and $\log(F/F_{MSY})$.

If $u = SSB/SSB_{\mathrm{MSY}}$ and $v = F/F_{MSY}$ then $x = \log(u)$ and $y = \log(v)$, and the VCM has the form:

$$VCM_{x,y} = \{\sigma_x^2 cov_{x,y} cov_y \sigma_y^2\} \quad (1)$$

where σ_x^2 is the variance of x, σ_y^2 is the variance of y and $cov_{x,y} = cov_{y,x}$ is the covariance of x and y.

However, the quantities required are not available on the log scale, Stock Synthesis outputs are Maximum Likelihood Estimates (MLE) of derived quantities, their asymptotic standard errors (SE) and the correlation between them.

The construction of the $VCM_{x,y}$ requires conducting normal to log-normal transformations. We approximate σ_x^2 and σ_y^2 as:

$$\sigma_x^2 = log\left(1 + \left(\frac{SE_u}{u}\right)^2\right) \quad \text{and} \quad \sigma_y^2 = log\left(1 + \left(\frac{SE_v}{v}\right)^2\right), \ (2)$$

where SE_u and SE_v are the asymptotic standard error estimate for $u = SSB/SSB_{\mathrm{MSY}}$ and $v = F/F_{MSY}$ respectively.

The covariance of x and y can then be approximated on log-scale by

$$cov_{x,y} = log\left(1 + \rho_{u,v}\right), \quad (3)$$

where $\rho_{u,v}$ denotes the correlation of u and v.

To generate the desired covariances between SSB/SSB_{MSY} F/F_{MSY} or other quantities, a multivariate random generator can be used i.e.

$$MVN(\mu_{x,y}, VCM_{x,y}), \ (4)$$

where $\mu_{x,y}$ is the vector of the MLEs of x and y. The joint MVLN distribution is then obtained as the exponential of the MVN.

2.2 Generating Kobe posteriors

The joint MVLN distribution of SSB/SSB_{MSY} and F/F_{MSY} can be obtained as described above using a multivariate random generator. For example, in R using the package '*mvtnorm*'.

SS3 outputs can be obtained using the ss_output() function in the R package 'r4ss' (Taylor *et al.,* 2013). The expected values of derived quantities are found in R the data frames *derived_quants*, and their covariances in *CoVar*.

R code used in this demonstration is available at https://github.com/laurieKell/SCRS-papers/tree/main/mvln The intention is to implement this in an R package in the future.

However, R code scripts that have used MVLN for the provision of management advice for several species may also be available separately within other SCRS working groups (E.g., Anon. 2021, their Section 9.1 BET – Bigeye Tuna; Anon. 2023b, their Section 9.3 SKJ – Skipjack and Section 9.12 SWO-AT – Atlantic swordfish).

Unfortunately, the availability of alternative R code scripts to implement MVLN can lead to confusion. For example, the availability of alternative R code scripts to implement MVLN within the 2023 blue shark assessment resulted in insufficient time at the 2023 blue shark stock assessment meeting to fully develop, then verify the code and validate the probabilistic advice based on simulation methods as described above (Anon 2023a). A good practice for reducing confusion in future ICCAT SCRC species group implementations of MVLN may be to unify the available R code scripts through a single github account, for example "ss3diags": https://github.com/PIFSCstockassessments/ss3diags. The ss3diags_handbook.Rmd provides example implementations of the functions 'SSplotEnsemble', `SSdeltaMLVN` and `SSdiagsMCMC` which easily implement many of the MVLN methods previously described for other ICCAT SCRS species (Walter *et al*. 2019; Winker *et al.* 2019; Walter and Winker 2020).

3. Preliminary results

The results present examples of the types of plots that help to understand stock dynamics and to summarise historical, current and future status.

3.1 Historical trends

Plots of historical trends and current status for SSB/SSB_{MSY} and F/F_{MSY} are shown in **Figures 1** and **2** respectively; medians, inter-quartiles and 90th percentiles are shown for the trends.

3.2 Kobe phase plots

Kobe phase plots, for F/F_{MSY} in 2020 and SSB/SSB_{MSY} in 2021, are shown in **Figure 3**, and for F/F_{MSY} in 2022 and SSB/B_{MSY} in 2022 in **Figure 4. Figure 5** shows Kobe phase plots for the years 2020 and 2022, correlations are also shown. The correlations are high for F/F_{MSY} in 2020, but decrease for 2021, and are small for 2022.

3.3 Projections

Deterministic projections for catch and target F's are shown in **Figure 6** and **Figure 7** for SSB/B_{MSY} and F/F_{MSY} respectively. The corresponding medians from the Monte-Carlo projections, including model error, are shown in **Figures 8** and **9.** The medians from the simulations are similar to the deterministic runs, there does appear to be a problem with the estimates of uncertainty for years when the stock is near to collapse, e.g. for TACs of 350 and 375.**Figure 10** shows the historical assessment and an F_{MSY} projection, along with two example Monte Carlo simulations. Note the serial correlations, these are important for projections that model feedback control.

3.4 Verification

For verification, i.e. as a check that the MLVN code produces the expected results, the covariances from stock synthesis and those simulated by MVLN and shown in **Tables 1** and **2**. The corresponding terminal estimates are shown in **Tables 3**, **4**, and **5**.

3.5 Kobe Strategy Matrices

The Kobe Strategy Matrix, i.e. the probability of being in the green Kobe quadrant, for a range of catch levels, is shown in **Table 6**. The corresponding probabilities of being overfished and of overfishing are shown in **Tables 7** and **8.**

3.6 Comparison of MVLN and MCMC

Figure 11 compares historical trends simulated by MVLN and preliminary MCMC results, ribbons show inter-quartiles and 90th percentile, relative to median; **Figure 12** compares Kobe Phase Plots in between F/F_{MSY} in 2021 and SSB/B_{MSY} in 2022. **Figures 13-16** present Kobe phase plots for preliminary MCMC results for alternative configurations of the forecast.ss file, which also affected correlations between F/F_{MSY} and SSB/B_{MSY} among years and, consequently, may benefit from additional investigation to ensure forecast.ss files are implemented consistently for MVLN and MCMC.

4. Discussion

Oft-stated reasons for not conducting long-term projections are that they are time-consuming to perform, or that results in the long term are more uncertain than those in the short term. This misunderstands the nature of projections, the aims for conducting them, and the difference between model types.

Using the delta-MLVN reduces the time taken to run the forecasts and makes it practical to conduct them during a stock assessment meeting. ICCAT routinely combines age and biomass-based models when providing advice. These may show different short-term trends due to process error, so short-term forecasts show more uncertainty than long-term forecasts. This is because in an age-based model process error is modelled by recruitment, i.e. year class abundance, while in a biomass-dynamic model, it is a random process on biomass. Therefore, age-based models may generate trends in the short term not seen in biomass-dynamic models, e.g. a stock may not recover even if fishing mortality is reduced. In the long term, the production function, and hence reference points, are determined by fixed parameters (Mangel *et al*., 2013) used to model age-specific processes or to develop priors. Therefore, the two types of models should converge, although the associated probabilities, and hence percentiles used for status relative to targets and limits, may differ. For these reasons, projections were run for 50 years into the future, to enable comparisons to be made with reference points derived from equilibrium assumptions. If the trajectories do not converge to the values associated with the production functions, then this may indicate an error in the code or model misspecification.

Conducting long-term forecasts also allows a comparison of the probabilities for reference points derived from the different models. The nature of the method used to estimate and propagate uncertainty is of particular importance for performing projections (Patterson *et al*., 2001). For example, when advice has to ensure that a stock is above or fluctuating around B_{MSY} or that there is a high probability of being above a limit reference point. Particularly, since MacCall (2013) showed that tail probabilities used to estimate status relative to limit reference points are unreliable. This is important since although targets based on medians from two different models may be similar, advice also depends on avoiding limits with high probability. In the latter case the tails of a joint probability may depend solely on a single model, e.g. a production model with priors for r, so when making statements such as there is only a 10% chance of being below B_{MSY} with a given F this may be wholly dependent on a given model choice.

A definition of a stock being at a level that supports MSY, includes fluctuating around B_{MSY} with no trend. This requires that the correct autocorrelation is simulated in the time series. Therefore, projections need to consider not just uncertainty in the current stock status but also on the nature of any serial correlations in the stock dynamic, for example, due to auto-correlation in recruitment and cohort effects (Bjørnstad *et al*., 2004). Therefore, the robustness of projection advice based on delta-MVLN and MCMC needs to be carefully evaluated.

5. Conclusions

The provision of fisheries management advice requires the assessment of stock status relative to reference points, as well as the prediction of the stock's response to management actions. Ideally, advice should follow the principle of "Risk Equivalence" to ensure that the probability of a stock being depleted below a limit reference point or not being maintained at a target reference point. Defined here as "Risk", is "Equivalent" irrespective of the stock assessment method, the amount of data and knowledge available, and the resulting uncertainty of the stock assessment method. This means that the methods used to estimate probabilities need to be comparable.

The main sources of uncertainty when providing stock assessment advice are i) parameters for which there is little information in the data and so are fixed in the model, e.g. natural mortality and steepness, ii) model structure, and iii) future process variability e.g. recruitment used in projections. However, inherent biases in the estimators, and biases due to model misspecification or quirks in the data flow through into the construction of the Kobe plot and Kobe strategy matrix (Maunder and Aires-da-Silva, 2011).

Although methods are available that directly include model structure uncertainty, e.g. Bayesian analyses using reversible jump MCMC, these are computationally intensive and so have not been used to date. Instead, ICCAT mainly uses sensitivity analysis to evaluate uncertainty by running integrated and biomass dynamic models and then varying fixed parameters or priors.

The construction of the Kobe Strategy Matrix for integrated assessments is computationally intensive if model structure uncertainty is taken into consideration. However, ignoring uncertainty in model structure uncertainty or naively including all model structures without appropriately weighting them can substantially bias the management actions presented in a Kobe Strategy Matrix for example. Particularly as management actions are sensitive to the model structure uncertainty. The use of the MVLN approximation method within the Kobe Strategy Matrix is therefore a practical alternative, but the accuracy of the approximation needs to be evaluated.

Comparing results from MVLN and MCMC is potentially a useful, yet time-consuming diagnostic, as differences may indicate problems with the assumed error distributions or model misspecification, but can also be a simple distortion from mixing informative Bayesian priors of key parameters into a maximum likelihood framework. The MVLN can be used to compare estimation and structural model error by plotting confidence intervals around point estimates obtained from a multimodel 'grid' approach. The major advantage of the MVLN approach over the bootstrap and MCMC routines is that it reduces the computing time from days to minutes. The results presented here suggest that the MLVN is a valid approach to represent uncertainty about the stock status that deserves further consideration, for example as summarised below.

The SCRS Shark Species Group has recommended that a more comprehensive comparison between MVLN and MCMC and bootstrap approaches be conducted among species within ICCAT and other tuna RFMOs by the ICCAT WGSAM (Anon. 2023b, their Section 17.1.3 Working Group on Stock Assessment Methods (WGSAM) Workplan (June 2023 – June 2024)). The SCRS Shark Species Group has also recommended considering hiring one or more external experts to assist in constructing a clear and comprehensive methodological approach to build an uncertainty grid for the 2024 stock assessment of Atlantic North and South shortfin mako shark stocks (Anon. 2023b, their Section 18.1.6 Sharks).

We have demonstrated here how probabilistic estimates can be provided in real-time for the current and future status of North Atlantic blue shark for possible use in a more comprehensive comparison between MVLN and MCMC and bootstrap approaches. The MVLN approach we demonstrate here can also be extended to build an uncertainty grid for the 2024 stock assessment of Atlantic North and South shortfin mako shark stocks.

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Table 1. Covariances from SS

	Bratio_2020	*Bratio_2021*	*Bratio_2022*	*F_2020*	*F_2021*	*F_2022*
Bratio_2020	1	0.98	0.93	-0.73	-0.56	-0.5
Bratio_2021	0.98	1	0.98	-0.78	-0.6	-0.54
Bratio_2022	0.93	0.98	1	-0.82	-0.63	-0.57
F_2020	-0.73	-0.78	-0.82	1	0.6	0.57
F_2021	-0.56	-0.6	-0.63	0.6	1	0.48
F_2022	-0.5	-0.54	-0.57	0.57	0.48	1

Table 2. Covariances simulated by MVLN

	Bratio_2020	*Bratio_2021*	*Bratio_2022*	*F_2020*	*F_2021*	*F_2022*
Bratio_2020	1	0.98	0.93	-0.74	-0.57	-0.52
Bratio_2021	0.98	1	0.98	-0.78	-0.6	-0.55
Bratio_2022	0.93	0.98	1	-0.81	-0.63	-0.57
F_2020	-0.74	-0.78	-0.81	1	0.61	0.59
F_2021	-0.57	-0.6	-0.63	0.61	1	0.5
F_2022	-0.52	-0.55	-0.57	0.59	0.5	1

Table 3. Summary of SS3 terminal estimates of $SS\,\frac{B}{B_{MSY}}$ and $\frac{F}{F_{MSY}}$

	2020	2021	2022	2023
Stock	1.063	1.0573	1.0329	0.9933
Harvest	0.775	0.7348	0.7582	0.7653

Table 4. Summary of MVLN median terminal estimates of $SS\,\frac{B}{B_{MSY}}$ and $\frac{F}{F_{MSY}}$

	2020	2021	2022	2023
Stock	1.0588	1.0558	1.0331	0.9941
Harvest	0.7785	0.7356	0.7578	0.7624

Table 5. Probability of being overfished, $P(SSB \leq B_{MSY})$, for different catch levels.

TAC	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
0 t	48	38	53	68	90	99	100	100	100	100
150 t	48	37	35	31	44	70	88	96	99	100
175 t	48	37	33	27	35	60	80	91	96	98
200 t	48	37	30	22	28	50	68	82	90	94
225 t	47	38	28	18	22	39	56	69	80	86
250 t	47	38	25	15	16	30	44	56	65	72
275 t	47	39	23	12	12	21	36	43	51	56
300 t	47	38	21	9	9	15	25	31	36	40
325 t	47	37	18	8	7	11	16	21	25	26
350 t	47	36	15	5	4	7	11	13	15	16
375 t	47	36	13	5	4	5	8	10	10	11
400 t	45	35	13	4	3	3	5	6	7	7

Table 6. Probability of over fishing, $P(F \geq F_{MSY})$, for different catch levels.

TAC	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
0 t	96	100	100	100	100	100	100	100	100	100
150 t	96	100	100	100	100	100	100	100	100	100
175 t	96	100	100	100	100	100	100	100	100	100
200 t	96	100	100	100	100	100	100	100	100	100
225 t	96	98	99	99	99	100	100	100	100	100
250 t	96	91	91	93	94	96	96	97	99	99
275 t	96	77	76	77	78	80	83	85	87	89
300 t	96	56	53	53	53	57	60	61	62	64
325 t	96	34	32	29	30	32	33	32	34	34
350 t	97	16	13	13	13	13	13	13	14	14
375 t	95	7	6	5	6	5	6	6	6	5
400 t	96	3	2	1	1	2	2	2	2	2

Table 7. Kobe II Strategy Matrix, $P(SSB \geq B_{MSY}) \cap P(F \leq F_{MSY})$.

TAC	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
0 t	48	38	53	68	90	99	100	100	100	100
150 t	48	37	35	31	44	70	88	96	99	100
175 t	48	37	33	27	35	60	80	91	96	98
200 t	48	37	30	22	28	50	68	82	90	94
225 t	47	38	28	18	22	39	56	69	80	86
250 t	47	38	25	15	16	30	44	56	65	72
275 t	47	38	23	12	12	21	35	43	51	56
300 t	47	32	19	9	9	15	24	30	34	38
325 t	47	23	13	7	6	9	14	18	21	21
350 t	46	12	7	4	3	5	8	9	10	11
375 t	47	6	4	2	2	3	4	5	5	5
400 t	45	2	1	1	1	1	1	2	1	2

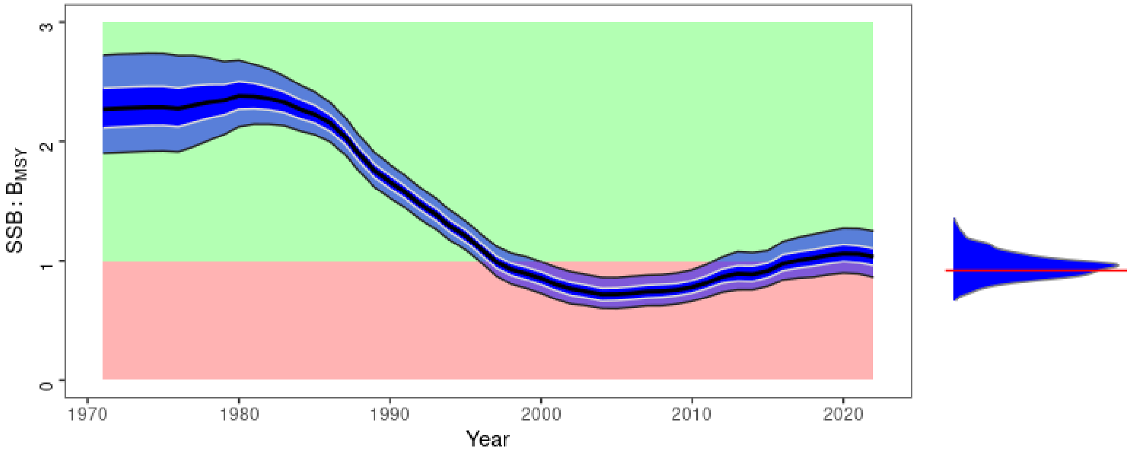

Figure 1. Historical trends in $SS\frac{B}{B_{MSY}}$ with marginal plot of current status, ribbons show inter-quartiles and 90_{th} percentile, relative to median.

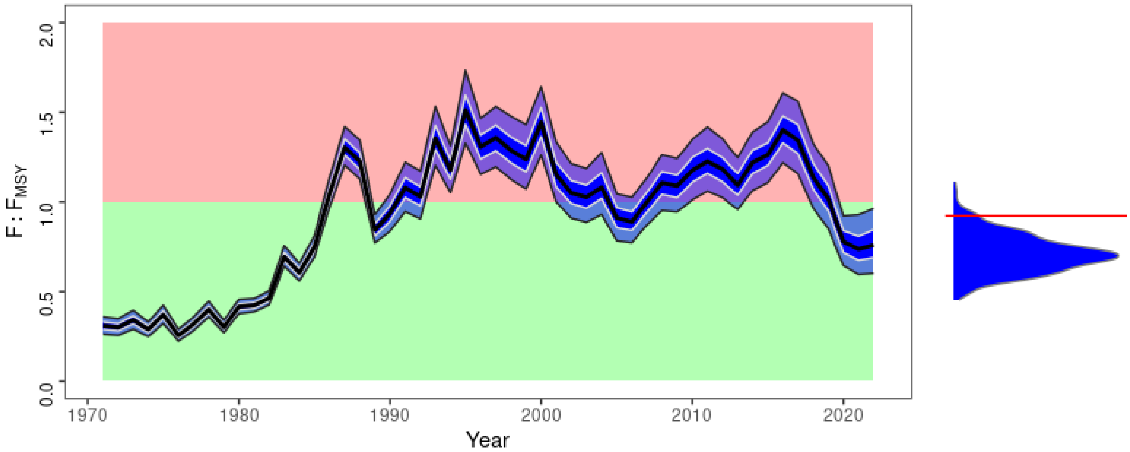

Figure 2. Historical trends in $\frac{F}{F_{MSY}}$ with marginal plot of current status, ribbons show inter-quartiles and 90_{th} percentile, relative to median.

Current Status

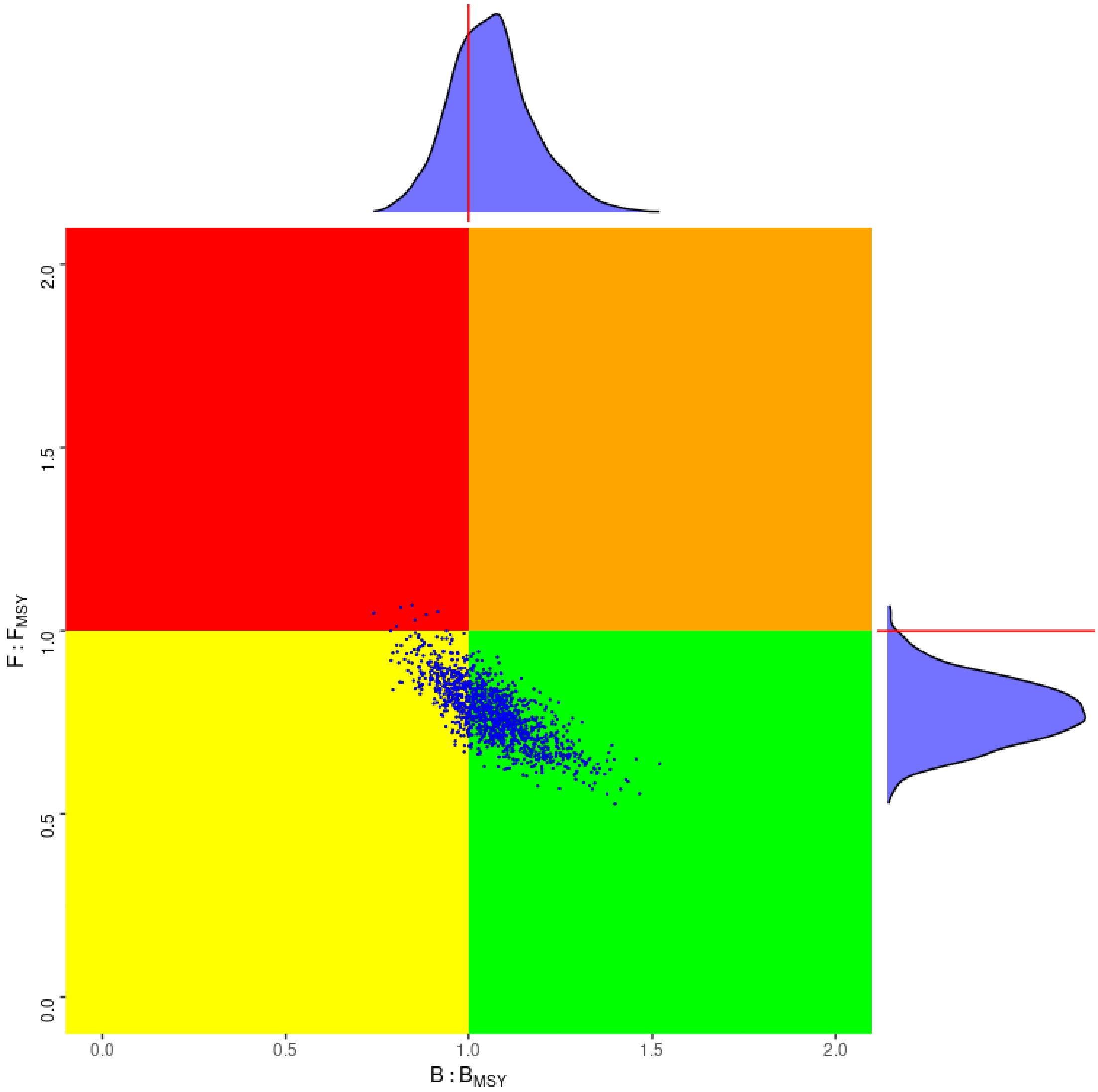

Figure 3. Kobe phase plot for $SS\,\frac{B}{B_{MSY}}$ in 2021 and $\frac{F}{F_{MSY}}$ in 2020

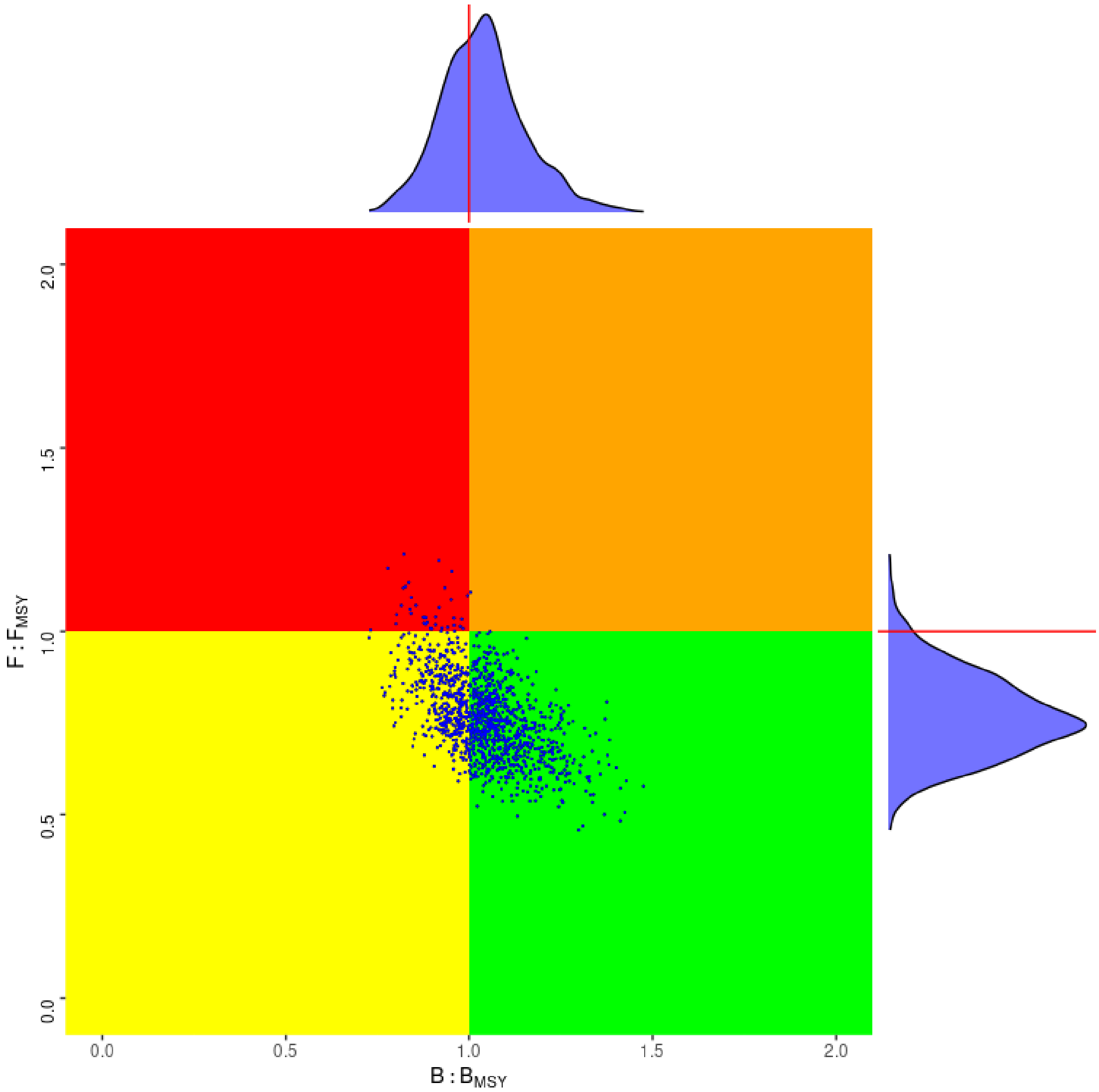

Figure 4. Kobe phase plot for $SS\,\frac{B}{B_{MSY}}$ in 2022 and $\frac{F}{F_{MSY}}$ in 2022

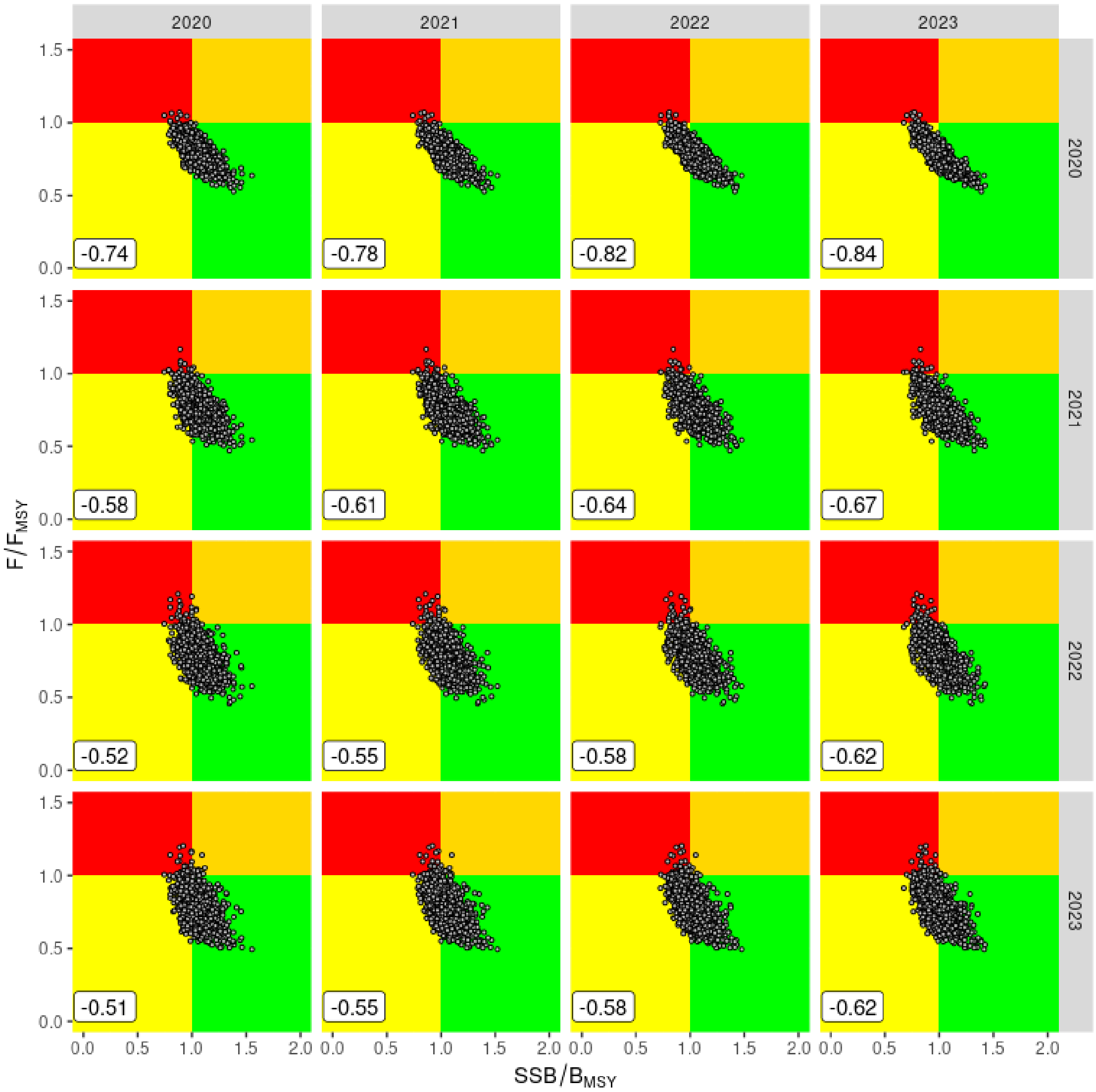

Figure 5. Kobe phase plots for historical and $\frac{F}{F_{MSY}}$ forecasts, with correlations.

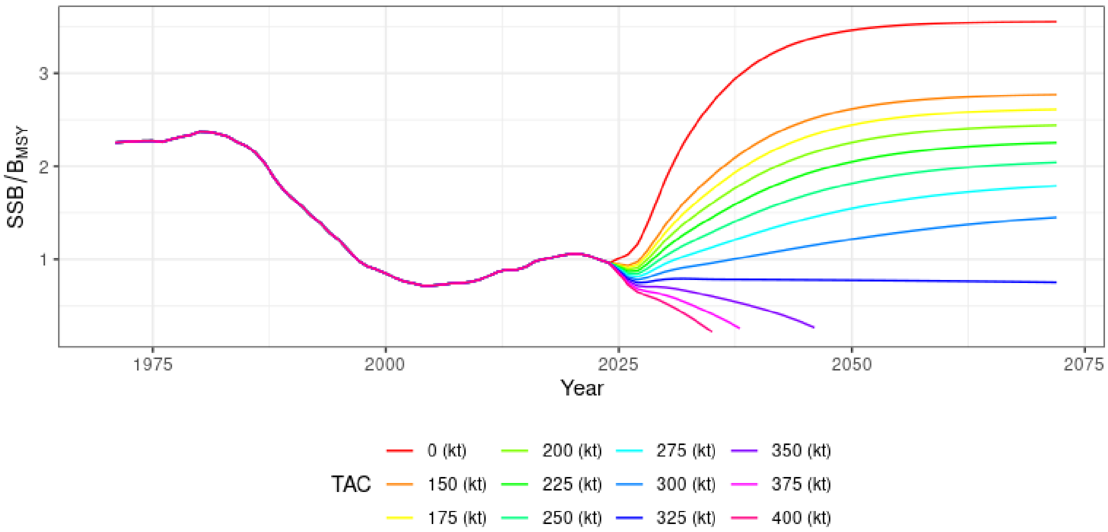

Figure 6. Deterministic values of $SS\frac{B}{B_{MSY}}$ for projections

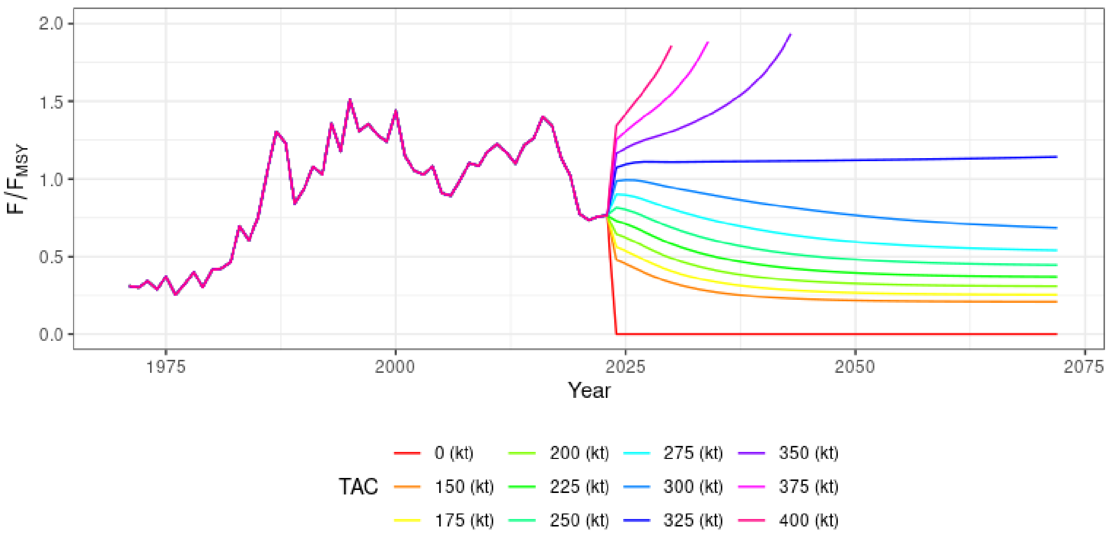

Figure 7. Deterministic values of $\frac{F}{F_{MSY}}$ for projections.

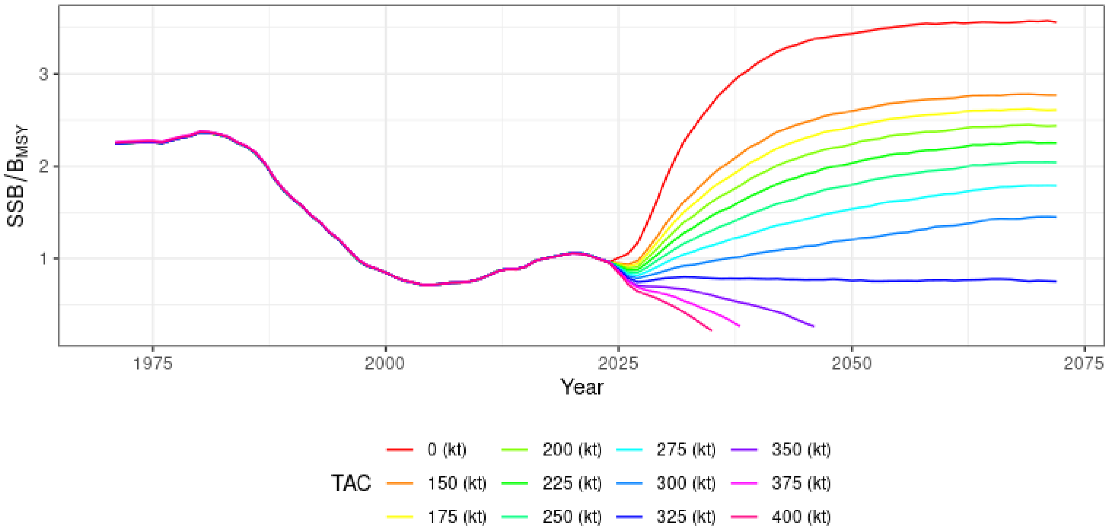

Figure 8. Median estimates of $\frac{B}{S}SB_{MSY}$ for projections.

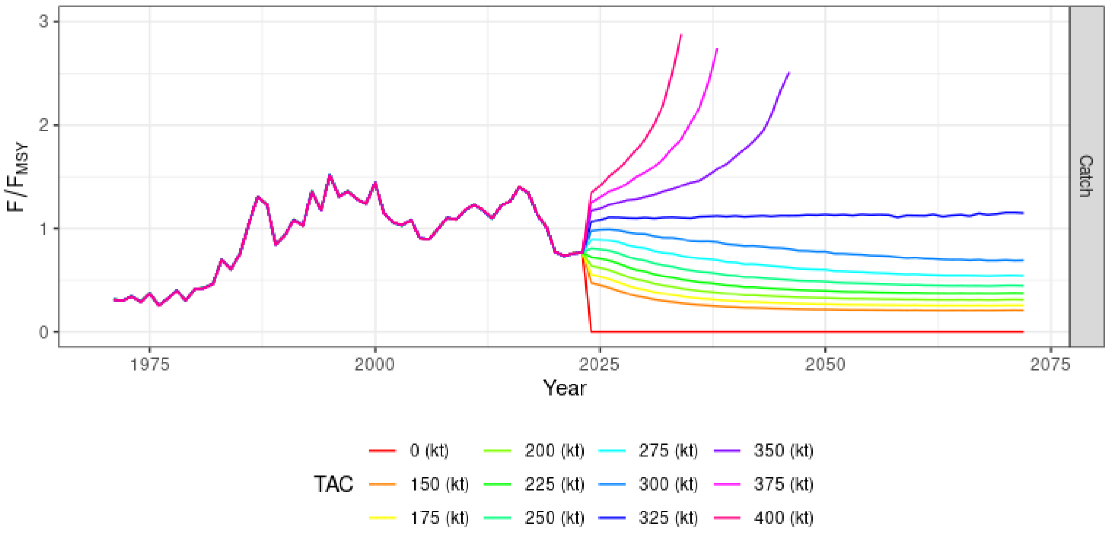

Figure 9. Median estimates of $\frac{F}{S}SB_{MSY}$ for projections.

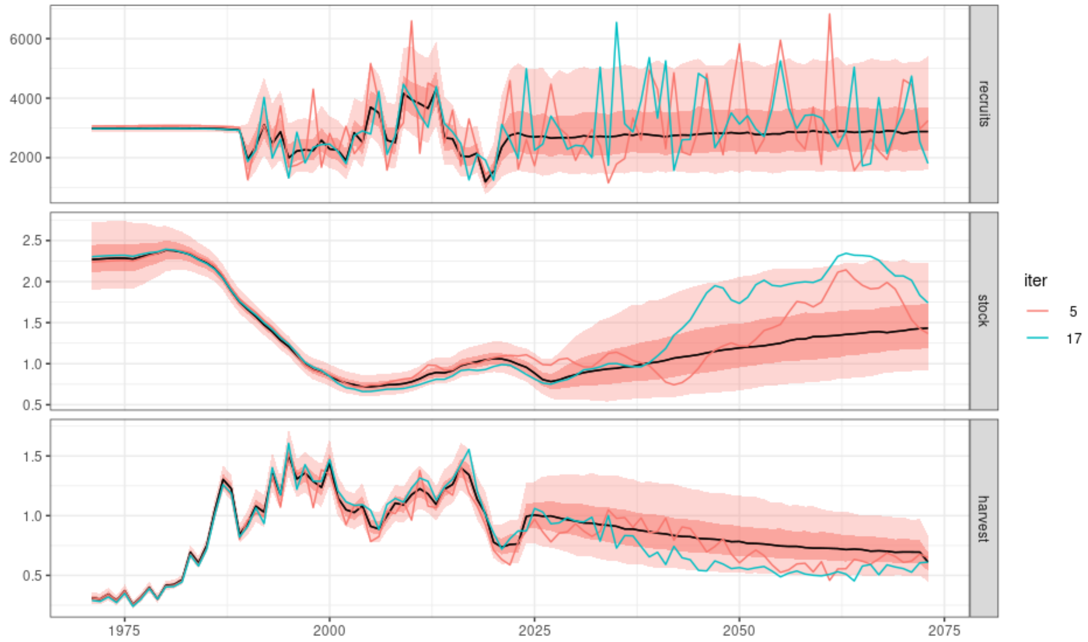

Figure 10. Plot of $\frac{B}{S}SB_{MSY}$ & $\frac{F}{F_{MSY}}$ with sample Monte Carlo simulations.

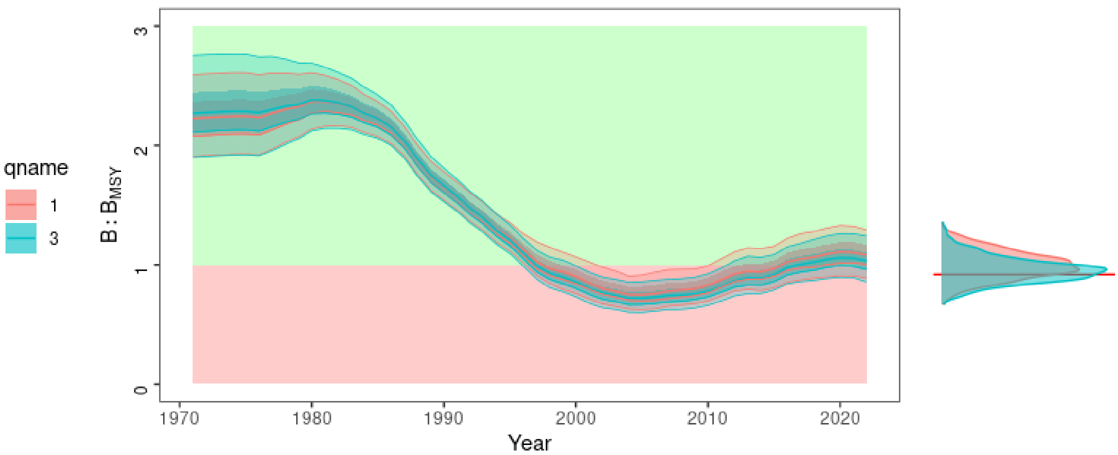

Figure 11. Comparison of MVLN and MCMC historical a trends in $SS\frac{B}{B_{MSY}}$, ribbons show inter-quartiles and 90_{th} percentile, relative to median.

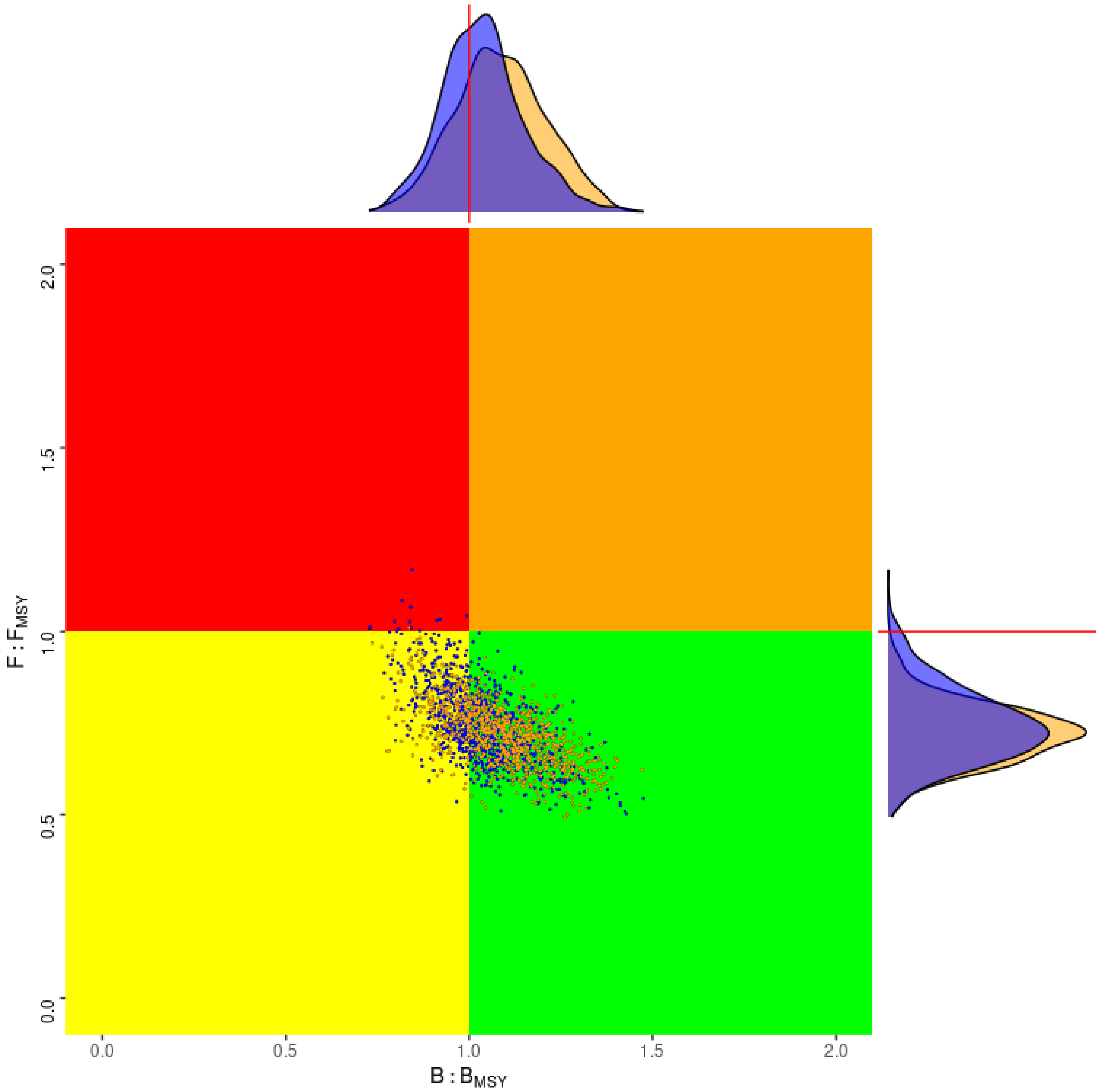

Figure 12. Comparison of MVLN and MCMC simulations for Kobe Phase Plot of of $\frac{B}{S}SB_{MSY}$ & $\frac{F}{F_{MSY}}$.

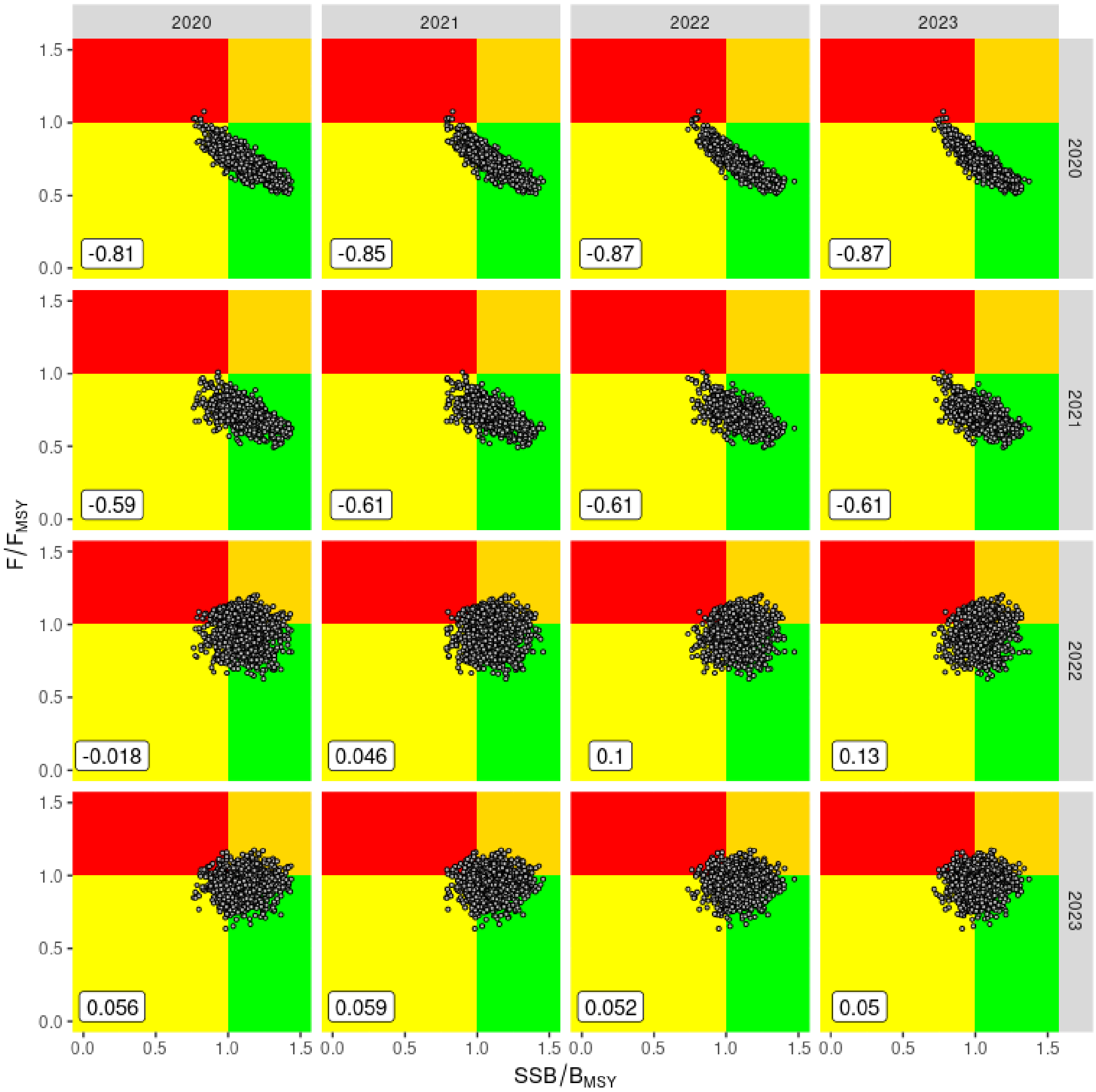

Figure 13. Kobe phase plots for MCMC, based on reference case model forecast.ss , with correlations between years.

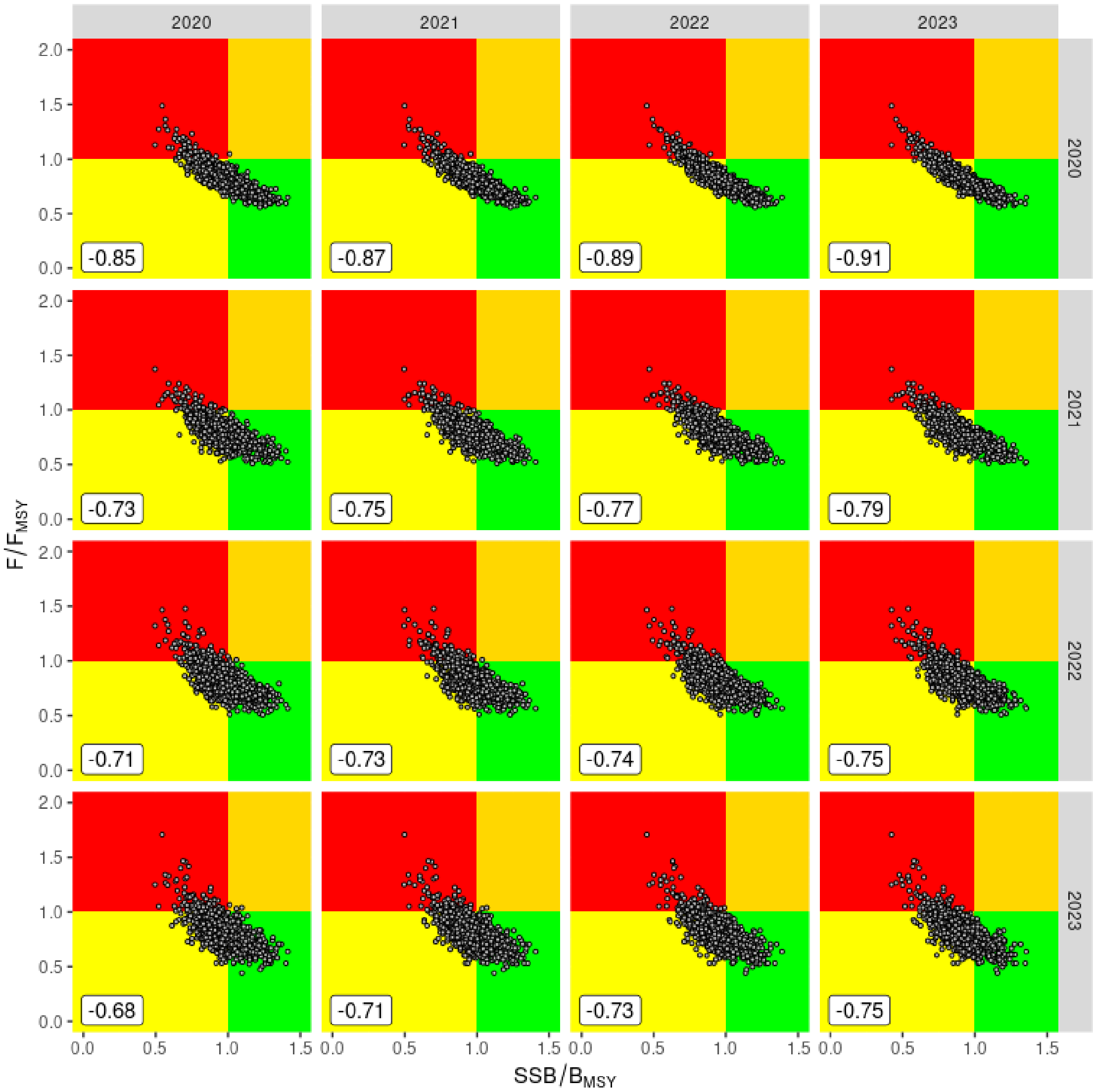

Figure 14. Kobe phase plots for MCMC, based on new forecast.ss, with correlations between years.

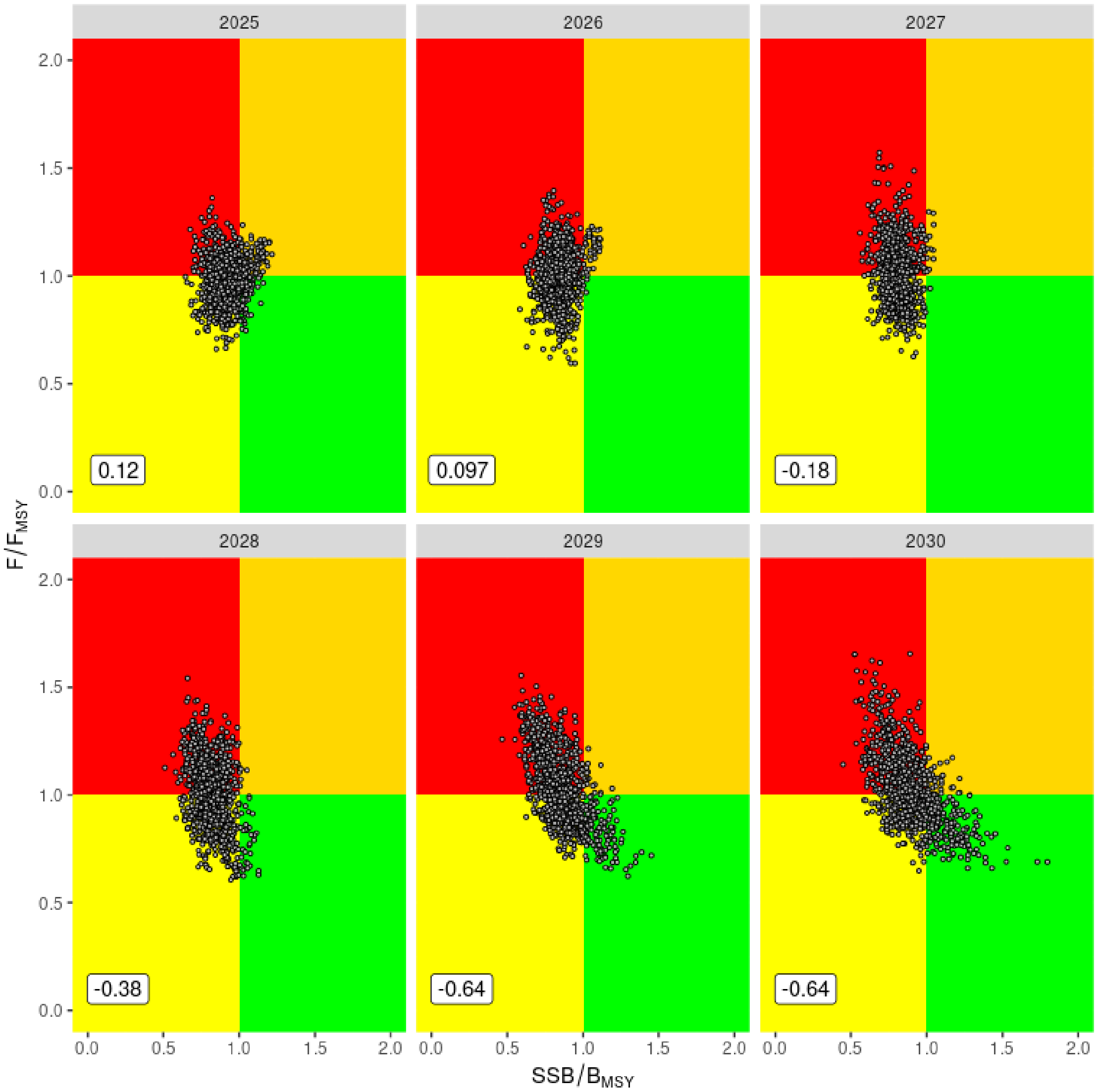

Figure 15. Kobe phase plots for MCMC, based on reference case model forecast.ss, with correlations between years.

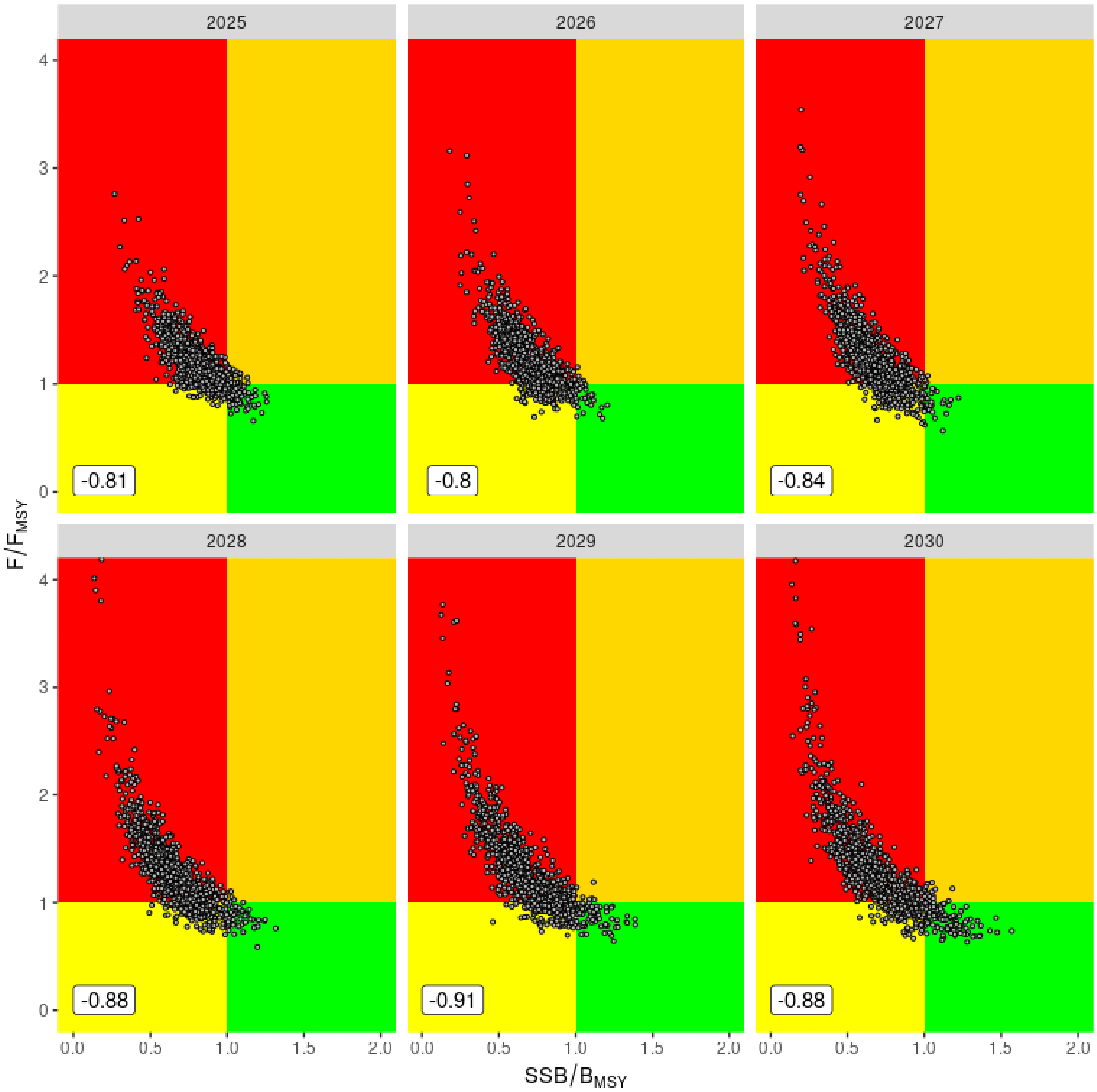

Figure 16. Kobe phase plots for MCMC, based on new forecast.ss, with correlations between years.