

ANALYSIS OF THE LENGTH-WEIGHT RELATIONSHIPS FOR THE ATLANTIC BLUEFIN TUNA

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SUMMARY

*The recently adopted models by the SCRS for the Atlantic bluefin tuna, *Thunnus thynnus* (L.) (RW= 0.0000159137 $SFL^{3.020584}$, WEST; and RW= 0.0000315551 $SFL^{2.898454}$, EAST), together with the models used to date (RW= 0.0000152 $SFL^{3.0531}$, for western stock; and RW= 0.000019607 $SFL^{3.0092}$, for eastern stock) are analyzed in using bi-variant samples (SFL (cm), RW (kg)) with the aim of validating them and establishing which model best fit the reality represented by the samples and, therefore, will have the greatest descriptive and predictive power. The result of the analysis indicates that the adopted models WEST and EAST clearly underestimates the weight of spawning ABFT. The result of the classical statistical analysis is confirmed by means of the quantile regression technique. Other biological and fisheries indicators also conclude that the models WEST and EAST gradually underestimates the weight of ABFT spawners (of 2–3 m) by 8–14%. The evolution of K throughout the year, by using the monthly L-W adopted models, does not represent the significant increase in weight that ABFT experiences in nature between August and December.*

RÉSUMÉ

*Les modèles récemment adoptés par le SCRS pour le thon rouge de l'Atlantique, *Thunnus thynnus* (L.) (RW= 0,0000159137 $SFL^{3.020584}$, OUEST; et RW= 0,0000315551 $SFL^{2.898454}$, EST), ainsi que les modèles utilisés jusqu'à présent (RW= 0,0000152 $SFL^{3.0531}$, pour le stock occidental; et RW= 0,000019607 $SFL^{3.0092}$, pour le stock oriental), sont analysés au moyen d'échantillons bivariés (SFL (cm), RW (kg)), dans le but de les valider et de déterminer le modèle qui correspond le plus à la réalité représentée par les échantillons, et qui aura par conséquent la puissance prédictive et descriptive la plus élevée. Les résultats de l'analyse indiquent que les modèles adoptés pour l'OUEST et l'EST sous-estiment clairement le poids des thons rouges reproducteurs. Les résultats de l'analyse statistique classique sont confirmés au moyen de la technique de régression quantile. D'autres indicateurs biologiques et de la pêche concluent également que les modèles OUEST et EST sous-estiment progressivement le poids des reproducteurs de thon rouge (de 2-3 m) à hauteur de 8 à 14 %. L'évolution de K tout au long de l'année, en utilisant les modèles L-W mensuels adoptés, ne représente pas l'augmentation significative du poids que les thons rouges de l'Atlantique connaissent à l'état sauvage entre août et décembre.*

RESUMEN

*Los modelos recientemente adoptados por el Comité Permanente de Investigación y Estadísticas (SCRS) para el atún rojo del Atlántico (ABFT), *Thunnus thynnus* (L.) (RW= 0.0000159137 $SFL^{3.020584}$, OESTE; y RW= 0.0000315551 $SFL^{2.898454}$, ESTE), junto con los modelos usados hasta la fecha (RW= 0.0000152 $SFL^{3.0531}$, para el stock occidental; y RW= 0.000019607 $SFL^{3.0092}$, para el stock oriental) y un modelo alternativo para el stock oriental (RW= 0.0000188 $SFL^{3.01247}$), se analizan usando muestras bivariantes (SFL (cm), RW (kg)) con el objetivo de validarlos y establecer qué modelo se ajusta mejor a la realidad representada por las muestras y, por tanto, tendrá el mayor poder descriptivo y predictivo. El resultado de los análisis indica que los modelos adoptados ESTE y OESTE claramente subestiman el peso del ABFT reproductor. El resultado del análisis estadístico se confirma por medio de la técnica de regresión por cuantiles. Otros indicadores pesqueros y biológicos concluyen también que*

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los modelos OESTE y ESTE subestiman gradualmente el peso de los reproductores de ABFT (de 2-3 m) en un 8-14%. La evolución de K durante el año, usando los modelos adoptados mensuales L - W , no representa el importante aumento en peso que el ABFT experimenta en la naturaleza entre agosto y diciembre.

KEYWORDS

*Atlantic bluefin tuna, Thunnus thynnus,
Length-weight relationships, quantile regression, condition factor (K)*

1. Introducción

Stock assessments made by the SCRS of ICCAT for the ABFT follow the designation of two separate stocks and apply a different length-weight relationship to each (ICCAT, 2010a; 2014); equation 1 (Parrack and Phares, 1979), for the western stock, and equation 2 (Arena, unpublished), in ICCAT (2006; 2010b), for the eastern stock.

$$RW = 0.0000152 SFL^{3.0531} \quad (1)$$

$$RW = 0.000019607 SFL^{3.0092} \quad (2)$$

Equations 1 and 2 have been questioned in recent times, and so several new equations have been proposed to the SCRS (equation 3, called *WEST*, and equation 4, called *EAST*), which are adaptations of those published by Rodríguez-Marín et al. (2013, 2014) in Rodríguez-Marín and Ortiz (2014). Models *WEST* and *EAST*, which were finally adopted without consensus by the ABFT stock assessment group in 2014 (ICCAT, 2014a; 2014b), have been recently published (Rodríguez-Marín et al., 2015).

$$RW = 0.0000159137 SFL^{3.020584} \quad (3)$$

$$RW = 0.0000315551 SFL^{2.898454} \quad (4)$$

Cort et al. (2015), and Cort and Estruch (2015) demonstrated that equations *EAST* and *WEST* clearly underestimates the weight of spawning ABFT up to 14%. This underestimation was possibly caused by the use of several data which have been converted from *CFL* to *FL*, because the conversion factor available in ICCAT is constant, while it should be logically variable according to length and other condition factors. This problem become much higher when the fish size is larger and when fish samples are related to months just before or during the spawning season. Higher the number of fish with conversion factor used for the equation, higher the possible bias.

In view of the above considerations, the specific aims of the present study are:

- i) To compare the values of the adopted models *WEST* and *EAST* (equations 3 and 4) with equations used to date (equations 1 and 2) to establish which model best represent the reality as represented by samples of ABFT spawners ($K \geq 2$) and, therefore, have the greatest descriptive and predictive power,
- ii) To check how the equations *WEST* and *EAST* adapt to the biology of ABFT by means of the growth curve, and other biological and fisheries indicators.

2. Material y Métodos

2.1 Western stock. Sample used and models subject to analysis

The sample used is based on data of spawners from the Gulf of Mexico (Knapp et al., 2010) and from fisheries of Canada (Caddy et al., 1976; Butler et al., 1977; Smith et al., 2006; Corrigan et al., 2007; Fraser, 2008 and Database from Fisheries and Oceans Canada). The sample contains a few young ABFT obtained from Rivas (1954), Baglin (1976); Farber and Chewing (1980); Hurley and Iles (1982), and own data from transatlantic migrations (East to West), cited in Cort (1990). In most of the sample (62%), size of the fish is referred to *SFL* but when fish are measured in *CFL* have been transformed using the formula $FL = 0.955 * CFL$ (Parrack et al., 1979).

The bi-variant sample used is the following:

- Longliners, Gulf of Mexico: $n= 29$; size range: 212–326 cm; sampling year: 2008 (March– May); $K= 1.86 \pm 0.24 SD$.
- Canadian fisheries: $n= 645$; size range: 150–320 cm; sampling years: 1975, 1976, 1999–2011 (July–October); $K= 2.03 \pm 0.23 SD$.

Data of young ABFT:

- Straits of Florida: $n= 2$; sizes: 25 and 45 cm; sampling years: 1951; 1953.
- USA Atlantic coast: $n= 22$; size range: 91–137 cm; sampling years: 1959, 1967, 1968, 1974, 1980, 1981 and 1982 (July–October).

Extreme data:

The sample contains two pairs of extreme data: two young-of-the-year of 25 cm (0.3 kg) and 45 cm (1.7 kg), sampled respectively in the Straits of Florida in November 1953 and January 1951 (Rivas, 1954); and two large spawners: one of 326 cm (655 kg) sampled in the Gulf of Mexico in 2008 (Knapp et al., 2010) and another 320 cm (679 kg), which is the ABFT sport fishing world record since 1979 (Fraser, 2008).

The set of the database ($n= 698$) will hereinafter be referred to as *GMX+CANADA* (Table 1A and Fig. 1A). The overall $K= 2.02 \pm 0.23 SD$

Models (equations) subject to analysis:

- Equation 3, from Rodríguez–Marín et al. (2013; 2014) and Rodríguez–Marín and Ortiz (2014) (hereafter, *WEST*):

$$RW= 0.0000159137 SFL^{3.020584} \text{ (hereafter, } \mathbf{WEST}) \\ n= 51,204$$

The model *WEST* is the last adaptation made by the two authors to the models published by Rodríguez–Marín et al. (2013; 2014).

- Equation 1, from Parrack and Phares (1979):

$$RW= 0.0000152 SFL^{3.0531} \text{ (hereafter, } \mathbf{Ec 1}) \\ n= 644$$

2.2 Eastern stock. Sample used, model presented and models subject to analysis

The sample used is based on samples from the Atlantic traps of Morocco (Abid et al., 2014 and INRH Database), Portugal (IPMA database, in Cort et al., 2013) and Spain (Rodríguez–Roda, 1967 and IEO database), and a number of samples from the database of the Program ICCAT–GBYP selected at random, which cover for the lack of lengths of young ABFT in the previous cases. In the three cases (traps of Morocco, Portugal and Spain) the mean value of K (Ricker, 1975) of the selected samples is $K > 2$.

The bi-variant sample used is the following:

- Data of Moroccan traps ($n= 278$); size range: 153–277 cm; sampling years: 1997, 2005, 2010–2013. Month of May; $K= 2.05 \pm 0.23SD$.
- Data of ICCAT–GBYP: $n= 196$; size range: 19–147 cm.

The set of the two databases ($n=474$) will hereinafter be referred to as *MOR+GBYP* (Table 1B and Fig. 1B).

The presented (alternative) model:

–Data of Spanish traps from Rodríguez–Roda (1967); $n=50$; size range: 127–251 cm; sampling year: 1963, months of May and 5th and 6th of June; $K=2.01 \pm 0.2SD$.

–Data of Spanish traps ($n=199$); size range: 127–284 cm; sampling years: 2001–2012, months of April and May; $K=2.02 \pm 0.19SD$ (IEO database).

–Data of Portuguese traps ($n=268$); size range: 120–278; sampling years: 1996–1998; 2000; 2007–2009 and 2011; $K=2.02 \pm 0.12SD$ (IPMA database, in Cort et al., 2013).

–Data of ICCAT–GBYP: $n=190$; size range: 19–130 cm.

The set of the four databases ($n=707$) will hereinafter be referred to as *ALM+GBYP*

Models (equations) subject to analysis:

- Equation 4 from Rodríguez–Marín et al. (2013; 2014) and Rodríguez–Marín and Ortiz (2014) (hereafter, *EAST*):

$$RW=0.0000315551 SFL^{2.898454}$$
$$n=74,096$$

Model *EAST* is the last adaptation made by the two authors to the models published by Rodríguez–Marín et al. (2013; 2014), a model based on a disproportionate number of ABFT due to the fact that approximately 60 % of the data set corresponded to specimens smaller than 2 m *SFL*.

- Equation 2 from Arena (ICCAT, 2010):

$$RW=0.000019607 SFL^{3.0092} \text{ (hereafter, Ec 2)}$$
$$n=\text{Unknown}$$

Model *Ec 2* is an adaptation of Arena's (1988) equation ($RW=0.0000178 SFL^{3.0283}$; $n=8,372$; Size range: 86–295 cm), a model mainly based on spawners caught in the Tyrrhenian Sea by Italian purse seiners (PS) in June, 1984–1988.

- Presented model (Cort et al., 2015):

$$RW=0.0000188 SFL^{3.01245} \text{ (ALM+GBYP; hereafter, Ec 3)}$$
$$n=707$$

2.3 Comparative validation study. Goodness of the fit, positional indicators and analysis of residuals

The two models were compared considering bi-variant samples (*SFL* (cm), *RW* (kg)) of 698 (*GMX+CANADA*), and 474 (*MOR+GBYP*) pairs of data to validate them and, therefore, establish which model best approximates the reality represented by the sample and to establish which one provides greater descriptive and predictive power.

For the validation of the models two samples were used (*GMX+CANADA* and *MOR+GBYP*) upon which the calculation of several indicators and statistical estimators has been made, establishing in all cases that a 95% confidence level was required. For a better and clearer interpretation of the results (case of *MOR+GBYP*), in the tables a progressive use of letters has been used for the models, from (a) to (d), which would indicate the ranking of the models from the best (a) to the worst (d), with intermediate qualifications (b) (worse than a) and (c) (worse than b but better than d).

Firstly, descriptive indicators were calculated: Coefficient of determination (R^2), mean absolute error, standard error of the absolute error, mean relative error and standard error of the relative error. In addition, the 95% confidence intervals have been calculated for the mean absolute error and for the mean relative error, which are robust estimates for the statistics described. The equi-distribution property was evaluated by calculating for each equation the percentages of real data that are above and below the curve and through the 95% confidence intervals for these proportions. Lastly, an analysis of the residuals was made for each model.

Mean of the absolute errors ($Eam = \frac{\sum |RW_i - Estimated RW_i|}{N}$ kg), Standard error of the absolute errors, (Standard deviation of the absolute errors / \sqrt{N}),

Mean of the relative errors ($Erm = \frac{\sum \frac{|RW_i - Estimated RW_i|}{Estimated RW_i}}{N} \cdot 100\%$) and Standard Error of the relative error (Standard deviation of relative error / \sqrt{N}).

2.4 Outliers

Outliers from *Fisheries and Oceans Canada* databases were removed based on the application of fixed values of Fulton's condition factor K (Ricker, 1975) between 1.4 and 2.6, according to Cort et al. (2013).

2.5. Quantile regression

With the aim of obtaining a more complete and robust analysis of the relationship between the variables length and weight and an approximate idea of the evolution of the distribution of weight as the ABFT grow in size, we resorted to the use of quantile regression (Koenker and Basset, 1978; Koenker, 2005), considering the data of the samples (*GMX+CANADA and MOR+GBYP*).

Taking into account the model $RW = a \cdot SFL^b$, the different curves corresponding to the selection of the quantiles 5%, 25%, 50%, 75% and 95% were obtained.

2.6 The fit of the equations to the growth equations of the western and eastern stocks, and to the weight of *GMX+CANADA and ALM+GBYP*. Estimation of K

The over or underestimation that may occur in the models studied was performed using the growth equations of the western ABFT stock ($Lt = 314.90 [1 - e^{-0.089(t+1.13)}]$), from Restrepo *et al.* (2010), and eastern stock ($Lt = 318.85 [1 - e^{-0.093(t+0.97)}]$), from Cort (1991) and Cort et al. (2014), in weight.

According to Gulland (1971), if the weight was proportional to the n power of the length, then the growth equation would be:

$$W_t = W_\infty [1 - e^{-k(t-t_0)^n}]$$

where:

W_t = Size (Weight, in kg) of the animal at time t (years)

W_∞ = Maximum mean asymptotic size (weight)

k = Growth rate (year^{-1})

t_0 = Theoretical age (years) moment at which $W = 0$

Using the growth equation, length (in cm) of a group of ages (5, 10, 15, 20, 25 and 30 years) was estimated and their corresponding value in weight (kg) applying the models studied.

The over or underestimation that may occur in models *WEST-Ec 1*, and *EAST-Ec 2*, *Ec 3*, was calculated from the residual analysis when comparing the different models. The study is based on the total weight of *GMX+CANADA* and *MOR+GBYP*.

To verify the fattening condition obtained when applying one or the other model (*WEST–Ec 1*, and *EAST–Ec 2*, *Ec 3*), the condition factor *K* (Ricker, 1975) has been calculated for the same values of size and weight/age, as in the previous case, and also for a wide range of length-weight values.

The evolution of *K* throughout the year is studied by means of the samples used in the present work. The results have been compared with *K* values obtained using the monthly *L-W* equations of Rodríguez-Marín et al. (2015) for a fix value of *SFL*= 220 cm.

In order to verify the relationship between *K* and the residual obtained by applying different length-weight equations various analyses have been made on samples whose size and actual weight are known. In each case the value of *K* and the residual obtained by applying the equations/month from Rodríguez-Marín et al. (2015) have been calculated. The same exercise regarding discarded equations of Parrack and Phares (1979), and Arena (in ICCAT, 2010) has been done. In the case of the equation of the eastern stock (Arena, in ICCAT, 2010) a sensitivity analysis, which does not include the month of December (*K* has a value between 2.3), was carried out.

3. Results

3.1 Study of comparative validation. Goodness of the fit, positional indicators and analysis of residuals

The models given by *Ec 1*, *Ec 2* and *Ec 3* have an overall fit to the data, significantly better than the model given by *WEST* and *EAST* if we consider the values of R^2 , the mean absolute error and the mean relative error (**Tables 2A** and **2B**). It can be observed how the upper ends of the 95% confidence intervals for absolute and relative errors corresponding to the equations *Ec 1*, *Ec 2* and remain below the lower ends of the respective intervals corresponding to the models *WEST* and *EAST* (**Tables 3A** and **3B**, and **Figs. 2A**, **2B** and **3A**, **3B**). Taking into account the goodness indicators of the fit described, the models given by *Ec 1*, *Ec 2* and *Ec 3* fit the data better and, in principle, will have greater predictive power than the equation *WEST* and *EAST*.

The results shown in **Table 4A** indicates that model *Ec 1* satisfies equi-distribution property (95% confidence level). The model *WEST* violate the property of equi-distribution underestimating weight. In the case of the model *WEST*, 84.24% of the real values are higher than the estimated values, which indicate that this model clearly underestimates weight. The model given by *Ec 1* overestimates the weight but only slightly.

On the other hand, in view of the results shown in **Table 4B** the models *EAST* and *Ec 2* violate the property of equi-distribution, with *EAST* underestimating weight and *Ec 2* overestimating it. In the case of the model *EAST*, 57.38% of the real values are higher than the estimated values, which indicate that this model clearly underestimates weight. The model given by *Ec 2* violates the equi-distribution property since the corresponding confidence intervals are the ones that are the farthest from the value 50. Model *Ec 3* satisfies equi-distribution property.

From the results of the analysis of the residuals (**Tables 5A** and **5B**; **Figs 4A** and **4B**), the difference between the mean and median values point to an important asymmetry of the residuals for the model *WEST* and *EAST* in comparison with models *Ec 1*, *Ec 2* and *Ec 3*, which can be checked visually in **Figures 5** and **6**. Models *Ec 1* and *Ec 3* strictly fulfills the requisite that the 95% confidence interval for the mean of the residuals contains the value 0. The 95% confidence interval for the residuals of *Ec 1* and *Ec 3* is the most accurate since, in addition to containing 0, it presents lower width, which means that it is a good predictive model with relatively low uncertainty. The mean values of the residuals are clearly lower, considering the absolute values, for models *Ec 1*, *Ec 2* and *Ec 3*. The positive and negative values of the mean (as well as the confidence intervals) for *WEST–EAST* and *Ec 1–Ec 2* confirm the tendency of models *WEST* and *EAST* to underestimate weight and models *Ec 1*, *Ec 2* overestimate weight, although the magnitude of these values would indicate that the predictive power of models *Ec 1* and *Ec 2* is greater than that of *WEST* and *EAST* (*Ec 1* and *Ec 2* overestimates weight but does so more slightly when compared with *WEST* and *EAST*, which underestimates it).

In view of all this, it can be concluded that the predictive models that would clearly (and plausibly) best explain the data of the samples are *Ec 1*, *Ec 2* and *Ec 3*. On the other hand, the models *WEST* and *EAST* would be evidently the least appropriate to explain the behaviour of the sample data.

3.3 Outliers

The used databases contain records of ABFT > 200 cm weighing < 25 kg ($K < 1$), as well as fishes < 200 cm with weights exceeding 350 kg ($K > 4$). Such type of records and other similar have been eliminated.

3.2 Quantile regression

Tables 6A and **6B** shows the results for the parameters provided by quantile regression for the quantiles selected, calculated from the samples *GMX+CANADA* and *MOR+GBYP*. As it can be seen in **Figures 7A** and **7B** the curves corresponding to *Ec 1* and *Ec 2* are slightly above the curve corresponding to the central quantile (50%) or median quantile. Models *WEST* and *EAST* are below the curve corresponding to quantile 25 and close to the one corresponding to quantile 5.

3.3 The fit of the equations to the growth equation of the western stock and to the weight of *GMX+CANADA*. Estimation of K

The evolution of K throughout the year is shown in **Figure 8** and **Table 7**. When using data from **Tables 10A** and **10B** three periods are observed: in the first, in July and August (after the reproduction), the ABFT is skinny; then there is a period of fattening that extends until the month of December; and finally, coinciding with the reproduction, there is a decline that extends until the month of May-June (blue squares). When using monthly L - W equations of Rodríguez-Marín et al. (2015) the values are very low, even not reaching the value 2 in any of the months (red squares).

Tables 8A and **8B** shows the result applying the growth equations (Restrepo et al., 2010 for the western stock, and Cort 1991; Cort et al., 2014 for the eastern stock). Firstly, the values of W_{∞} that is obtained on applying the equations *WEST* and *EAST* (559 kg; 570 kg) are unreal values very far from the actual world record (679 kg; Fraser, 2008), or from the official value of W_{\max} (726 kg; in ICCAT, 2010b). The W_{∞} obtained by applying *Ec 1*, *Ec 2* and *Ec 3* (644 kg; 670 kg; 655 kg) are much more realistic. When comparing the results of the two models, models *WEST* and *EAST* underestimates weight as ABFT ages increase (see **Tables 8A** and **8B**).

With the total sample weight of *GMX+CANADA* being **207,940** kg, the residuals obtained through the application of the *WEST* and *Ec 1* models have been as follows:

- Weight of the sample applying *WEST* model: 183,193 kg (-11.9%)
- Weight of the sample applying *Ec 1* model: 209,420 kg (0.7%)

With the total sample weight of *MOR+GBYP* being **61,410** kg, the residuals obtained through the application of the *EAST*, *Ec 2* and *Ec 3* models have been as follows:

- Weight of the sample applying *EAST* model: 55,560 kg (-9.4%)
- Weight of the sample applying *Ec 2* model: 62,585 kg (1.9%)
- Weight of the sample applying *Ec 3* model: 61,071 kg (-0.5%)

The obtained result is the same as that of the previous exercise, applying the growth equation. In the last column of **Tables 8A** and **8B** it is verified that the values of K obtained by applying the *WEST* and *EAST* models represent fish in low fattening condition with K very far from 2, while those obtained applying the *Ec 1*, *Ec 2* and *Ec 3* models are clearly fish in high fattening condition ($K \geq 2$).

The results in **Table 9A** are also very conclusive, verifying that for a wide range of size-weight values, the average value of K obtained using the *WEST* model ($K = 1.77$) represents fish in low fattening condition, while when applying the *Ec 1* model, the value of $K = 1.99$ is for fish in high fattening condition. With respect **Table 9B**, although the overall value (1.89) is higher than that in the previous case, actually spawners fish (> 140 cm *SFL*) have values of K ($= 1.82$), very far from $K = 2$.

Figure 9A shows the relationship between the value of K , in each of the samples in **Tables 10A** and **10B**, and the residual obtained for each of them by applying the monthly length-weight equations published by Rodríguez-Marín et al. (2015). The results clearly show how the residual increases as K also increases in such a way that it can be predicted by the regression line what the residual will be from the value of K . When the fish are in low fattening condition the residual is very small (< 5%), while for fish in high fattening condition it can reach 23%, implying that the adopted equations, from Rodríguez-Marín et al. (2015), are much better adapted to fish in low fattening condition.

Figures 9B and **9C** represent the same exercise as the above case but using discarded equations of Parrack and Phares (1979) and Arena (in ICCAT, 2010). The result is the opposite; i.e., when the fish are in low fattening condition the residual is very high (up to 22%), while fish are in high fattening condition residual is <5%. That is to say that the discarded models are much better adapted to fish in high fattening condition. However, when the value of K is very high Arena's equation do not fit either and has a high residual, as it is the case of December, for the eastern stock ($K = 2.3$; the residual is 11%).

The figures presented in the Photo Gallery show different morphological aspects of the ABFT, according to its value of K .

Discussion

The outliers are of great importance in obtaining a model by least square methods because their inclusion or not in the sample can influence widely the fitting parameters.

Although the data recognized as outliers could be possible, the relationship between the variables defined by the fitted model can be greatly affected by the outliers which may introduce important bias in the model that disappears if the data that should be treated as exceptions are eliminated in the data set.

In conclusion, if our goal is to fit a model that represents the usual behavior of the relationship between length and weight, singular data may introduce undesirable bias in the final model predictions. If we do not consider the possible but unusual data for constructing the model, the model will represent the usual behavior and the initial outliers will continue being outliers in reference to the values predicted by the model.

The results obtained from the various analyzes performed, allows us to confirm that the models predictive that would clearly best explain the data of the samples (*GMX+CANADA*; *MOR+GBYP*), from a statistical point of view, are *Ec 1*, *Ec 2* and *Ec 3*, whereas models *WEST* and *EAST* would not be appropriate to explain the behaviour of the data.

If a sample (SFL , RW) is homogeneous and representative, except in exceptional cases, it will present a high degree of symmetry, which will be manifested in the curve corresponding to quantile 50, $C50$, which corresponds to the evolution of the median, appearing quite centered when compared, on one hand with $C25$ and $C75$, and also if compared with $C5$ and $C95$ (**Figures 7A** and **7B**). In this case the curve obtained by simple least squares regression can be expected, which best explains the evolution of the mean, to appear close to $C50$, which is clearly observed for *Ec 1*, *Ec 2* and *Ec 3* (**Figures 7A** and **7B**). In the case of the curves *WEST* and *EAST*, the separation with respect to the median confirms what was concluded in the previous statistical analysis, which is that *Ec 1* and *Ec 2* slightly overestimates the representative central value of the weight, and *WEST* and *EAST* clearly underestimates it. It can be said that, based on the sample considered, *WEST* and *EAST* would only be representative of the length-weight relationship for tunas below the 25 % percentile of weight for one size.

Moreover, in view of the results of W_{∞} obtained on fitting the growth equation to the models *WEST-Ec 1* and *EAST-Ec 2*; *Ec 3*, it is concluded that *Ec 1*, *Ec 2* and *Ec 3* represents the biology of ABFT growth much better, and it can therefore be applied perfectly well to ABFT juveniles and spawning adults. Moreover, and as conclusive proof of its authenticity, models *Ec 1*, *Ec 2* and *Ec 3* satisfies the criterion that for $RW = 725$ kg (W_{max}), $SFL = 319.93 \pm 11.3$ cm (L_{max}), in accordance with Cort et al. (2013; 2014); this is not true for the models *WEST* and *EAST*.

The important disagreements found when applying the *WEST-EAST*, and *Ec 1*, *Ec 2* and *Ec 3* models regarding the real weight of the samples *GMX+CANADA* and *MOR+GBYP* confirm, through different methodologies, that the *WEST* and *EAST* models significantly underestimates the real weight of ABFT up to 14%.

According to Rodríguez-Roda (1964); Santos *et al.* (2004); Aguado and García (2005); Chapman *et al.* (2011), values of K between 1.4–1.7 are values for wild ABFT in a low fattening condition, far from what spawning ABFT have ($K \geq 2$), as has been demonstrated by: Rodríguez-Roda (1964); Percin and Akyol (2009; 2010); Golet and Lutcavage, unpublished data cited by Chapman *et al.* (2011); Deguara *et al.* (2012), Gordo (2010) and Galaz (2012).

The study of the evolution of K throughout the year (**Figure 8**; values in **Table 7**) confirms what was said in the preceding paragraphs; namely that the L - W equations adopted (Rodríguez-Marín *et al.*, 2015) underestimate the real weight of the ABFT. Moreover, it is observed in this particular case that when such equations are used the evolution of K throughout the year does not represent the significant increase in weight that ABFT experiences in nature between August and December.

The results of **Tables 9A** and **9B** (K column) using values of size and average weight/age are sufficiently important to confirm that the *WEST* and *EAST* models represent fish in low fattening condition, while *Ec 1*, *Ec 2* and *Ec 3* models represent fish in high fattening condition. The same result is obtained when applying both models to a wide range of size-weight values (**Tables 9A** and **9B**).

Figures 9A, **9B** and **9C** (values in **Tables 10A** and **10B**) show that adopted models (*WEST* and *EAST*), as well as the adopted monthly equations (Rodríguez-Marín *et al.*, 2015), represent fish in low fattening condition. It can be observed that when samples have a low K (mean value of $K \leq 1.8$) the residual is very small (<5%), while when fish have high-very high K (mean value of $K > 2.0$ to 2.4) the residual can reach 23%. On the contrary, the discarded equations (Parrack and Phares, 1979, western stock and Arena, in ICCAT, 2010, eastern stock) represent fish in high fattening condition. In these two cases, the residual is very high when samples have a low K (mean value of $K \leq 1.8$) while when K is high-very high (mean value of $K > 2.0$ to 2.4) the residual is very small.

The results obtained in the present study statistically prove that there are significant differences between the models adopted by the SCRS over three decades ago (*Ec 1*, *Ec 2*) and the model presented in this paper (*Ec 3*), representing the three of them the spawning population of ABFT which adapts to the growth parameters of this species, and two others that does not adapt and which represents the population of ABFT in low fattening condition (*WEST* and *EAST*). Therefore, it should be noted that the utilization of the length weight models adopted by the SCRS in 2014 for the western and eastern stocks, which underestimates the true weight of the ABFT (between 2–3 m) up to 14% (*WEST* and *EAST*), can greatly impact results in future ABFT stock assessments.

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Table 1.a) Summary statistics of the sample *GMX+CANADA*.b) Summary statistics of the sample *MOR+GBYP*.

A)			B)		
	<i>SFL</i> (cm)	<i>RW</i> (kg)		<i>SFL</i> (cm)	<i>RW</i> (kg)
Count	698.0	698.0	Count	474	474
Mean	240.1	297.9	Mean	161.654	129.557
Median	247.0	299.5	Median	193	151
Standard deviation	37.2	110.5	Standard deviation	69.4228	105.955
Minimum	25.0	0.3	Minimum	19	0.126
Maximum	326.0	679.0	Maximum	277	384
Range	301.0	678.7	Range	258	383.874
Lower quartile	229.0	232.0	Lower quartile	83	11
Upper quartile	264.0	377.0	Upper quartile	220	218

Table 2.

a) Descriptive indicators of the goodness of fit of the equations to the data.

b) Descriptive indicators of the goodness of fit of the equations to the data.

A)

	R^2 (%)	Mean of the absolute errors (MAE) (kg)	Standard error of the absolute errors (kg)	Mean of the relative errors (MRE) (%)	Standard error of the relative errors (%)
<i>WEST</i>	86.2	39.32	1.25	15	0.42
<i>Ec 1</i>	90.0	27.68	0.81	9.33	0.24

B)

	R^2 (%)	Mean of the absolute errors (MAE) (kg)	Standard error of the absolute errors (kg)	Mean of the relative errors (MRE) (%)	Standard error of the relative errors (%)
<i>EAST (c)</i>	95.28	14.67	0.81	10.18	0.43
<i>Ec 2 (b)</i>	96.67	11.93	0.7	7.94	0.27
<i>Ec 3 (a)</i>	96.93	11.38	0.7	7.6	0.29

Table 3.

95% confidence intervals for the mean of the absolute errors (MAE) and for the mean of the relative errors (MRE).

A)

	CI (95%) MAE	CI (95%) MRE
<i>WEST</i>	39.3195 +/- 2.4502 [36.8693; 41.7697]	15.0034 +/- 0.8256 [14.1778; 15.829]
<i>Ec 1</i>	27.6818 +/- 1.5847 [26.0971; 29.2665]	9.3275 +/- 0.4720 [8.85546; 9.79947]

B)

	CI (95%) MAE	CI (95%) MRE
<i>EAST (c)</i>	14.6721 +/- 1.59869 [13.0734; 16.2708]	10.179 +/- 0.837 [9.342; 11.017]
<i>Ec 2 (b)</i>	11.9303 +/- 1.37316 [10.5571; 13.3034]	7.944 +/- 0.523 [7.421; 8.467]
<i>Ec 3 (a)</i>	11.3853 +/- 1.3224 [10.0629; 12.7077]	7.572 +/- 0.564 [7.008; 8.136]

Table 4.

Positional indicators to assess whether the models provide estimated values of the weight higher or lower than the actual values.

A)

	Percentage of actual values lower than estimates values (%)	95% confidence intervals for the percentage of actual values lower than estimated values.
	(Percentage of actual values higher than estimated values (%))	(95% confidence intervals for the percentage of actual values higher than the estimated values.)
<i>WEST</i>	15,76	[13.1347; 18.6785]
	(84. 24)	([81.3215; 86.8653])
<i>Ec 1</i>	53,72	[49.9403; 57.4682]
	(46. 28)	([42.5318; 50.0597])

B)

	Percentage of actual values lower than estimates values (%)	95% confidence intervals for the percentage of actual values lower than estimated values.
	(Percentage of actual values higher than estimated values (%))	(95% confidence intervals for the percentage of actual values higher than the estimated values.)
<i>EAST (c)</i>	42.62	[38.1208; 47.2118]
	(57. 38)	([52.7882; 61.8792])
<i>Ec 2 (d)</i>	65. 61	[61.1411; 69.8827]
	(34. 39)	([30.1173; 38.8589])
<i>Ec 3 (b)</i>	54.64	[50.0356; 59.1861]
	(45. 36)	([40.8139; 49.9644])

Table 5.

Summary statistics for the residuals corresponding to the different models analyzed, with respect to the global data and confidence intervals (95%) for the average of the residuals.

A)

	<i>WEST</i>	<i>Ec 1</i>
Count	698	698
Mean	35.4744	-2.0999
Standard deviation	37.1325	34.9182
Median	28.619	-2.9779
95% confidence interval for the mean	35.4744 +/- 2.7547 [32.7197; 38.2291]	-2.09994 +/- 2.59044 [-4.69038; 0.490505]

B)

	<i>EAST (d)</i>	<i>Ec 2 (b)</i>	<i>Ec 3 (a)</i>
Count	474	474	474
Mean	11.7863	-2.939	0.26334
Standard deviation	19.7551	19.1167	18.5609
Median	0.4584	-0.6877	-0.1664
Confidence interval (95.0%) for the mean	11.7863 +/- 1.783 [10.0033; 13.5693]	-2.93903 +/- 1.72538 [-4.66441; -1.21365]	0.2634 +/- 1.6752 [-1.41189; 1.93856]

Table 6.

- A) Results for the parameters provided by the quantile regression for the selected quantiles calculated from the sample *GMX+CANADA* (in Figure 7A).
- B) Results for the parameters provided by the quantile regression for the selected quantiles calculated from the sample *MOR+GBYP* (in Figure 7B).

A)

Percentile curve/parameter	a	b
5%	1.7379E-05	2.993942
25%	2.0184E-05	2.993847
50% (median)	1.7913E-05	3.020156
75%	1.9340E-05	3.022508
95%	1.4997E-05	3.087066

B)

Percentile curve/parameter	a	b
5%	2.00787E-05	2.968608
25%	1.94296E-05	2.993847
50% (median)	1.82497E-05	3.015722
75%	1.66805E-05	3.049626
95%	1.58711E-05	3.079667

Table 7.

Evolution of *K* throughout the year obtained by means of the samples shown in Tables 10A and 10B (■). Last two columns, evolution of *K* throughout the year and weight corresponding to a fixed value of *SFL*= 220 cm (■), both obtained by using monthly *L-W* adopted equations (Rodríguez-Marín et al., 2015). See Figure 8

Western and Eastern Atlantic							K-ICCAT ■	W (kg), 220 cm
	K ■	Max	Min	Stand Dev.	N	Observations		
July (E)	1.86	2.06	1.660	0.20	127	Traps, Hnd (S. of Gibraltar)	1.75	187
July (W)	1.83	1.93	1.730	0.10	57	Canada	1.70	181
August (E)	1.83	1.97	1.690	0.14	144	Traps (S. of Gibraltar)	1.76	187
August (W)	1.88	2.05	1.710	0.17	135	Canada	1.75	187
September (E)	2.01	2.10	1.920	0.09	16	North Sea (Spor)	1.79	191
September (W)	2.14	2.34	1.940	0.20	153	Canada	1.79	190
October (W)	2.21	2.43	1.990	0.22	122	Canada	1.85	197
November (W)	2.30	2.61	1.990	0.31	29	Canada	1.82	194
December (E)	2.32	2.51	2.130	0.19	51	Morocco (Hnd line)	1.79	191
February (E)	1.99	2.25	1.730	0.26	39	Traps (S. of Gibraltar)	1.80	191
April (E)	2.00	2.16	1.840	0.16	91	Traps (S. of Gibraltar)	1.83	195
May (E)	1.89	2.18	1.600	0.29	211	Traps (S. of Gibraltar)	1.90	201
April/May (W)	1.84	2.07	1.610	0.23	25	Gulf of Mexico (LL)	1.83	196
June (E)	1.78	1.95	1.610	0.17	115	Traps (S. of Gibraltar)	1.87	199

Table 8.

- a) Comparison of the estimated size (*SFL*, cm), round weight (*RW*, kg) and *K* at age obtained from the models *WEST* and *Ec 1* applied to the growth equation of the western ABFT stock.
- b) Comparison of the estimated size (*SFL*, cm), round weight (*RW*, kg) and *K* at age obtained from the models *EAST*, and *Ec 2* and *Ec 3* applied to the growth equation of the eastern ABFT stock.

A)

a		<i>SFL</i> (cm)	<i>RW</i> (kg)	Diference, a/b (%)	<i>K</i>
$Lt = 314.90 [1 - e^{-0.089(t+1.13)}]$					
$W = 0.0000159137 L^{3.020584}$ (<i>WEST</i>)					
$Wt = 559 [1 - e^{-0.089(t+1.13)}]^{3.020584}$					
	Age 5	132	40	-11.1	1.74
	Age 10	198	137	-12.2	1.76
	Age 15	240	246	-12.4	1.78
	Age 20	267	340	-12.6	1.79
	Age 25	284	409	-13.0	1.79
	Age 30	295	459	-13.1	1.79
b		<i>SFL</i> (cm)	<i>RW</i> (kg)	Diference, a/b (%)	<i>K</i>
$Lt = 314.90 [1 - e^{-0.089(t+1.13)}]$					
$W = 0.0000152 L^{3.0531}$ (<i>Ec 1</i>)					
$Wt = 644 [1 - e^{-0.089(t+1.13)}]^{3.0531}$					
	Age 5	132	45	–	1.96
	Age 10	198	156	–	2.01
	Age 15	240	281	–	2.03
	Age 20	267	389	–	2.04
	Age 25	284	470	–	2.05
	Age 30	295	528	–	2.06

B)

a		<i>SFL</i> (cm)	<i>RW</i> (kg)	Diference, a/b (%)	<i>K</i>
$Lt = 318.85 [1 - e^{-0.093(t+0.97)}]$					
$W = 0.0000315551 L^{2.898454}$ (<i>EAST</i>)					
$Wt = 570 [1 - e^{-0.093(t+0.97)}]^{2.898454}$					
	Age 5	136	48	-7.7	1.91
	Age 10	204	156	10.9	1.84
	Age 15	247	272	-12.5	1.81
	Age 20	273	363	-13.6	1.78
	Age 25	290	433	-13.9	1.78
	Age 30	301	482	-14.3	1.77
b		<i>SFL</i> (cm)	<i>RW</i> (kg)	Diference, a/b (%)	<i>K</i>
$Lt = 318.85 [1 - e^{-0.093(t+0.97)}]$					
$W = 0.000019607 L^{3.0092}$ (<i>Ec 2</i>)					
$Wt = 670 [1 - e^{-0.093(t+0.97)}]^{3.0092}$					
	Age 5	136	52	–	2.07
	Age 10	204	175	–	2.06
	Age 15	247	311	–	2.06
	Age 20	273	420	–	2.06
	Age 25	290	503	–	2.06
	Age 30	301	563	–	2.06
c		<i>SFL</i> (cm)	<i>RW</i> (kg)	Diference, a/c (%)	<i>K</i>
$Lt = 318.85 [1 - e^{-0.093(t+0.97)}]$					
$W = 0.0000188 L^{3.01247}$ (<i>Ec 3</i>)					
$Wt = 655 [1 - e^{-0.093(t+0.97)}]^{3.0092}$					
	Age 5	136	50	-4.0	1.99
	Age 10	204	170	-8.2	2.00
	Age 15	247	303	-10.2	2.01
	Age 20	273	410	-11.4	2.02
	Age 25	290	492	-12.0	2.02
	Age 30	301	550	-12.4	2.02

Table 9.

Condition factor (*K*) calculated for a wide range of length-weight values, using models *WEST* and *Ec 1* (A), and models *EAST*, *Ec2* and *Ec 3* (B).

A)

Model	<i>WEST</i>		<i>Ec 1</i>	
<i>a</i>	1.59137E-05		0.0000152	
<i>b</i>	3.020584		3.0531	
<i>SFL</i> (cm)	<i>W</i> (kg)	<i>K</i> (<i>WEST</i>)	<i>W</i> (kg)	<i>K</i> (<i>Ec 1</i>)
40	1.1	1.72	1.2	1.85
60	3.7	1.73	4.1	1.89
80	8.9	1.74	9.8	1.92
100	17.5	1.75	19.4	1.94
120	30.3	1.76	33.9	1.96
140	48.3	1.76	54.2	1.98
160	72.4	1.77	81.5	1.99
180	103.3	1.77	116.8	2.00
200	142.0	1.77	161.1	2.01
220	189.3	1.78	215.5	2.02
240	246.3	1.78	281.1	2.03
260	313.6	1.78	358.9	2.04
280	392.3	1.79	450.1	2.05
300	483.2	1.79	555.6	2.06
320	587.2	1.79	676.6	2.06
Mean (<i>K</i>)		1.78	Mean (<i>K</i>)	1.99
<i>SD</i>		± 0.02	<i>SD</i>	± 0.06

B)

Model	<i>EAST</i>		<i>Ec 2</i>		<i>Ec 3</i>	
<i>a</i>	3.15551E-05		0.000019607		0.0000188	
<i>b</i>	2.898454		3.0092		3.01247	
<i>SFL</i> (cm)	<i>W</i> (kg)	<i>K</i> (<i>EAST</i>)	<i>W</i> (kg)	<i>K</i> (<i>Ec 2</i>)	<i>W</i> (kg)	<i>K</i> (<i>Ec 3</i>)
40	1.389	2.17	1.3	2.03	1.3	1.97
60	4.497	2.08	4.4	2.04	4.3	1.98
80	10.354	2.02	10.5	2.04	10.2	1.99
100	19.769	1.98	20.5	2.05	19.9	1.99
120	33.534	1.94	35.4	2.05	34.5	2.00
140	52.423	1.91	56.3	2.05	54.9	2.00
160	77.199	1.88	84.1	2.05	82.0	2.00
180	108.611	1.86	119.9	2.06	117.0	2.01
200	147.401	1.84	164.7	2.06	160.7	2.01
220	194.301	1.82	219.4	2.06	214.1	2.01
240	250.036	1.81	285.1	2.06	278.3	2.01
260	315.325	1.79	362.7	2.06	354.2	2.01
280	390.881	1.78	453.3	2.07	442.7	2.02
300	477.410	1.77	557.9	2.07	545.0	2.02
320	575.614	1.76	677.5	2.07	662.0	2.02
Mean (<i>K</i>)		1.82	Mean (<i>K</i>)	2.05		2.00

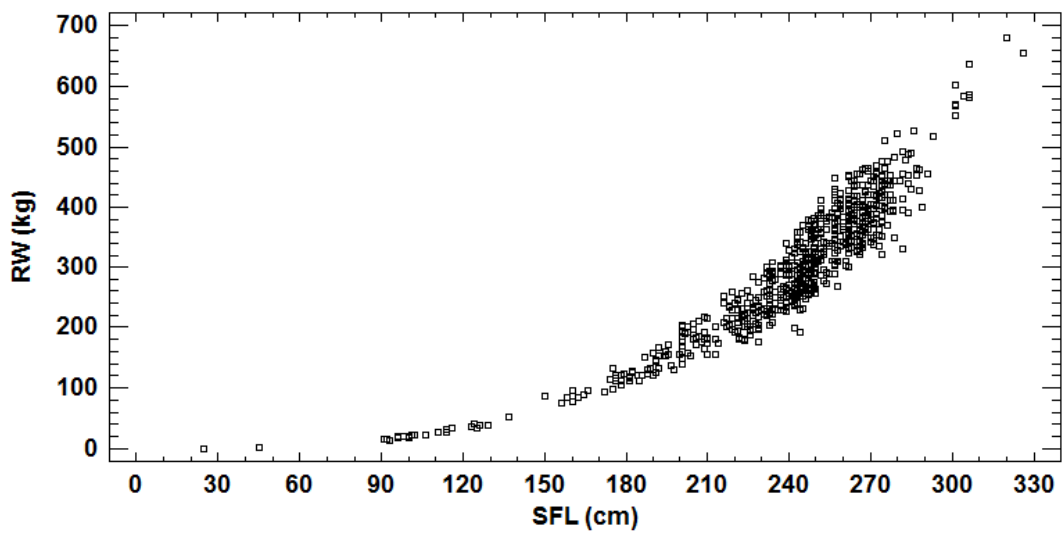
Table 10A. Samples used in the present study.

WESTERN STOCK												
Number	Source	Fish measured and weighted (<i>n</i>)	Size range (cm)	Sampling year/s	Month	<i>K</i> factor	Real weight (kg)	Inferred weight (kg) by using equation 1	Equation 1 (Source)	Inferred weight (kg) by using equation 2	Equation 2 (Source)	Observations
1	Knapp et al. (2010)	25	213-261	2008	April-May	1.84±0.23SD	6,341	6,066	$RW = 0.00001771 SFL^{2.598823106}$ (R.-Marín et al., 2015)	6,214	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	LPRL database, Gulf of Mexico
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-4.3	-	-2.0	-	-
2	Butler et al. (1977)	57	233-278	1975, 1976	July	1.83±0.10SD	17,067	15,861	$RW = 0.00001771 SFL^{2.59824736}$ (R.-Marín et al., 2015)	16,805	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-7.1	-	-1.5	-	-
3	Butler et al. (1977)	48	235-293	1975, 1976	August	2.04±0.17SD	18,439	15,894	$RW = 0.00001771 SFL^{2.59823761}$ (R.-Marín et al., 2015)	17,108	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-13.8	-	-7.2	-	-
4	Smith et al. (2006)	29	209-291	2005	August	1.79±0.18SD	7,443	7,368	$RW = 0.00001771 SFL^{2.59823761}$ (R.-Marín et al., 2015)	7,947	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-1.0	-	6.8	-	-
5	Corrigan et al. (2007)	58	191-289	2006	August	1.82±0.17SD	16,339	15,757	$RW = 0.00001771 SFL^{2.59823761}$ (R.-Marín et al., 2015)	16,987	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-3.6	-	4.0	-	-
6	Butler et al. (1977)	153	189-244	1975, 1976	September	2.14±0.20SD	60,684	50,993	$RW = 0.00001771 SFL^{3.00172394}$ (R.-Marín et al., 2015)	58,305	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-16.0	-	-3.9	-	-
7	Butler et al. (1977)	17	150-279	1975, 1976	October	2.33±0.18SD	6,772	5,422	$RW = 0.00001771 SFL^{3.00724859}$ (R.-Marín et al., 2015)	6,637	$RW = 0.00003871 SFL^{3.3172}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-19.9	-	-2.0	-	-
8	F&Oceans Canada Db	105	155-292	2008, 2009	October	2.09±0.27SD	33,417	28,940	$RW = 0.00001771 SFL^{3.00724859}$ (R.-Marín et al., 2015)	32,919	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-10.7	-	-	-	-
9	Butler et al. (1977)-F&O Canada Db	29	188-281	1975, 1983, 1984, 1993, 2006, 2011	November	2.3±0.31SD	9,816	7,776	$RW = 0.00001771 SFL^{3.00492886}$ (R.-Marín et al., 2015)	9,522	$RW = 0.00003871 SFL^{3.3172}$ (Parrack & Phares, 1979)	Canada
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-20.8	-	-2.9	-	-
10	Cort & Estruch (2016)	698	25-326	1951, 1953, 1959, 1967, 1968, 1974, 1976, 1980-82, 1999-2011, 2008	April-November (adults); June-October (juveniles)	2.02±0.23SD	207,940	183,193	$RW = 0.0000159137 SFL^{3.020584}$ (R.-Marín et al., 2014a,b)	209,420	$RW = 0.0000152 SFL^{3.08531}$ (Parrack & Phares, 1979)	General data
	Residuals/ <i>L</i> - <i>W</i> s (%)	-	-	-	-	-	-	-11.9	-	-0.7	-	-

Table 10B. Samples used in the present study.

EASTERN STOCK												
Number	Source	Fish measured and weighted (n)	Size range (cm)	Sampling year/s	Month	K factor	Real weight (kg)	Inferred weight (kg) by using equation 1	Equation 1 (Source)	Inferred weight (kg) by using equation 2	Equation 2 (Source)	Observations
1	Abid et al. (2014)	39	84-244	2007, 2009	February	1.99±0.26SD	6,296	5,786	$RW = 0.00003508 SF_L^{2.87528971}$ (R.-Marin et al., 2015)	6,614	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	INRH database. Atlantic traps, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-8.1	-	5.1	-	-
2	IEO database	91	163-274	2008-2011	April	2.0±0.16SD	20,880	19,219	$RW = 0.00003508 SF_L^{2.8763824}$ (R.-Marin et al., 2015)	21,715	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Atlantic traps, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-8.0	-	4.0	-	-
3	IEO database	211	142-275	2010	May	1.89±0.29SD	33,605	35,654	$RW = 0.00003508 SF_L^{2.88091328}$ (R.-Marin et al., 2015)	38,382	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Atlantic traps, S. of Gibraltar. "Arrival run"
	Residuals/ L-Ws (%)	-	-	-	-	-	-	6.1	-	14.2	-	-
4	IEO database + R. Roda, 1967	115	171-259	1963, 2013	June	1.78±0.17SD	20,156	21,198	$RW = 0.00003508 SF_L^{2.88209179}$ (R.-Marin et al., 2015)	23,326	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Atlantic traps + Hand line, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	5.2	-	15.7	-	-
5	Abid et al. (2014)	80	125-270	1997, 2010	July	2.02 ±0.27SD	9,815	8,896	$RW = 0.00003508 SF_L^{2.8715331}$ (R.-Marin et al., 2015)	10,011	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	INRH database. Hand line, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-9.4	-	2.0	-	-
6	Rodriguez-Roda (1967)	47	189-258	1963	July	1.71±0.14SD	8,093	8,334	$RW = 0.00003508 SF_L^{2.87153307}$ (R.-Marin et al., 2015)	9,772	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Atlantic traps, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	3.0	-	20.7	-	-
7	Rodriguez-Roda (1967)	22	133-243	1963	August	1.75±0.09SD	3,312	3,372	$RW = 0.00003508 SF_L^{2.87220195}$ (R.-Marin et al., 2015)	3,924	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Atlantic traps, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	1.8	-	18.5	-	-
8	Abid et al. (2014)	96	144-263	2002, 2010, 2012	August	1.9±0.22SD	13,582	12,949	$RW = 0.00003508 SF_L^{2.87220195}$ (R.-Marin et al., 2015)	14,993	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	INRH database. Hand line, S. of Gibraltar
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-4.7	-	10.4	-	-
9	Russell, 1934	26	220-258	1933	August	1.83±0.11SD	6,535	6,202	$RW = 0.00003508 SF_L^{2.87220195}$ (R.-Marin et al., 2015)	7,353	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	North Sea
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-5.1	-	12.5	-	-
10	Russell, 1934; Ross, 2010	16	209-265	1930, 1933; 1948, 1949	September	2.01±0.09SD	4,731	4,146	$RW = 0.00003508 SF_L^{2.87577309}$ (R.-Marin et al., 2015)	4,830	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	North Sea
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-12.4	-	2.1	-	-
11	Rey & Cort, unpublished	51	172-262	1979	December	2.32±0.19SD	13,454	10,318	$RW = 0.00003508 SF_L^{2.87522487}$ (R.-Marin et al., 2015)	11,937	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	Cape Sim (Morocco)
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-23.3	-	-11.3	-	-
12	Cort et al. (2015)	474	19-277	1997, 2005, 2010-2013	May (adults); June-October (juveniles)	2.10±0.20SD	61,410	55,823	$RW = 0.0000315551 SF_L^{2.898454}$ (R.-Marin et al., 2014a,b)	63,005	$RW = 0.000019607 SF_L^{3.0092}$ (Arena, in ICCAT, 2010)	General data
	Residuals/ L-Ws (%)	-	-	-	-	-	-	-9.1	-	2.6	-	-

A)



B)

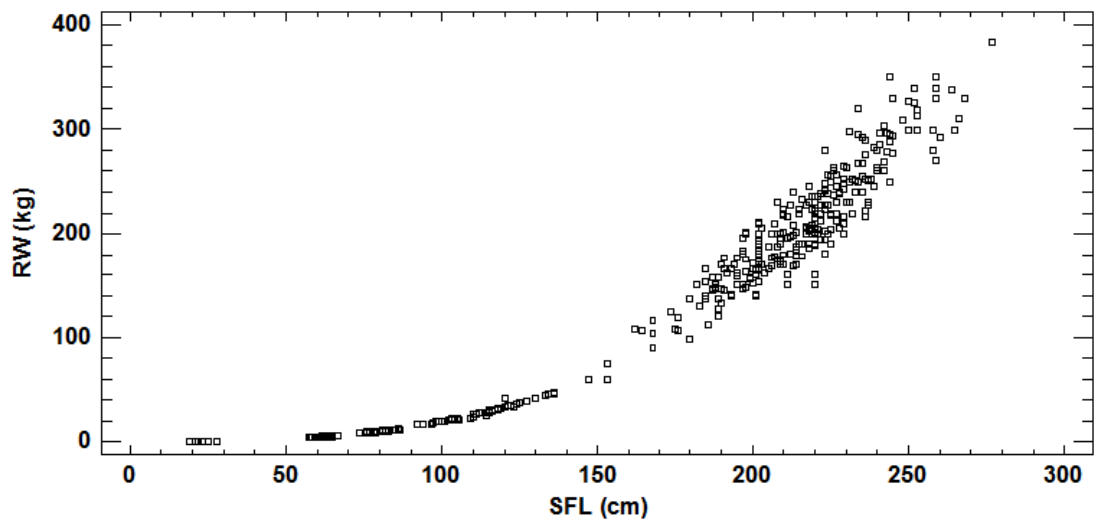


Figure 1. A) Plot of the used sample (*GMX+CANADA*); B) Plot of the used sample (*MOR+GBYP*).

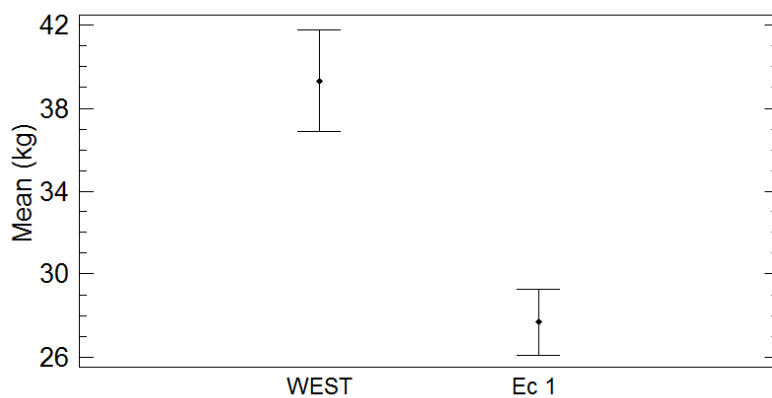


Figure 2. A. Means and 95% confidence intervals for the absolute errors.

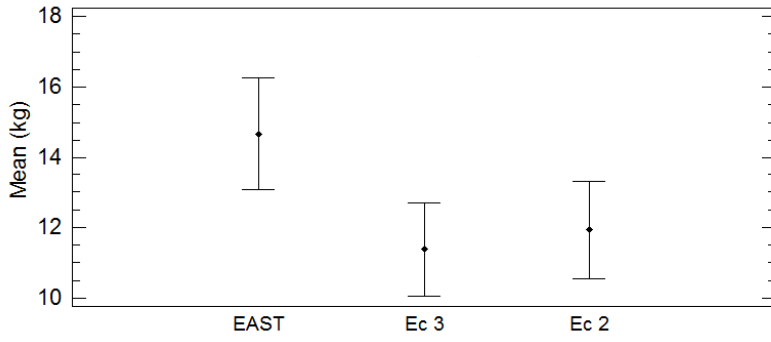


Figure 2B. Means and 95% confidence intervals for the absolute errors.

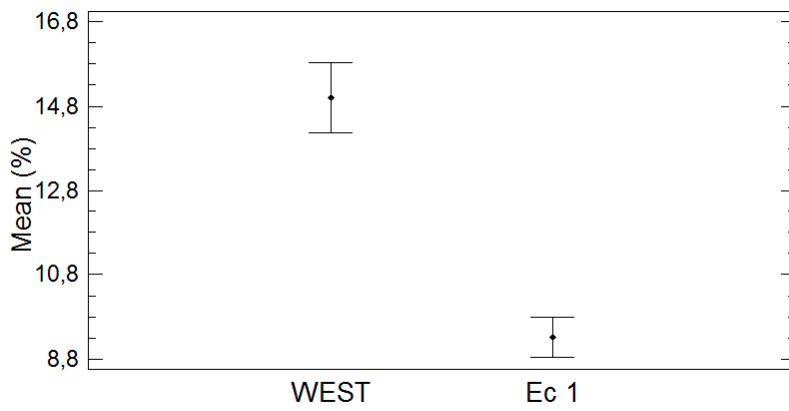


Figure 3A. Means and 95% confidence intervals for the relative errors.

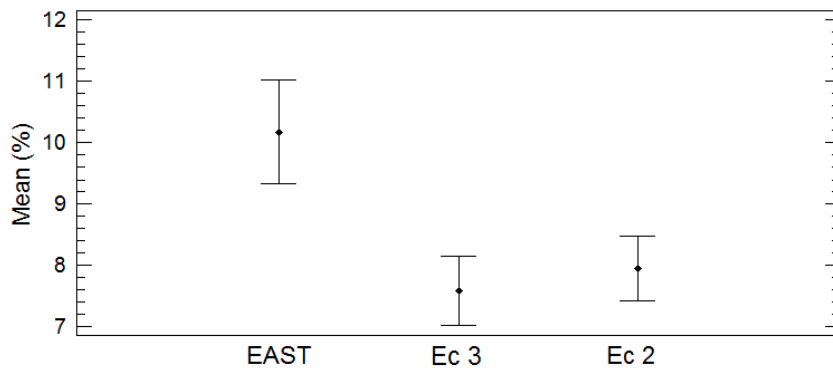


Figure 3B. Means and 95% confidence intervals for the relative errors.

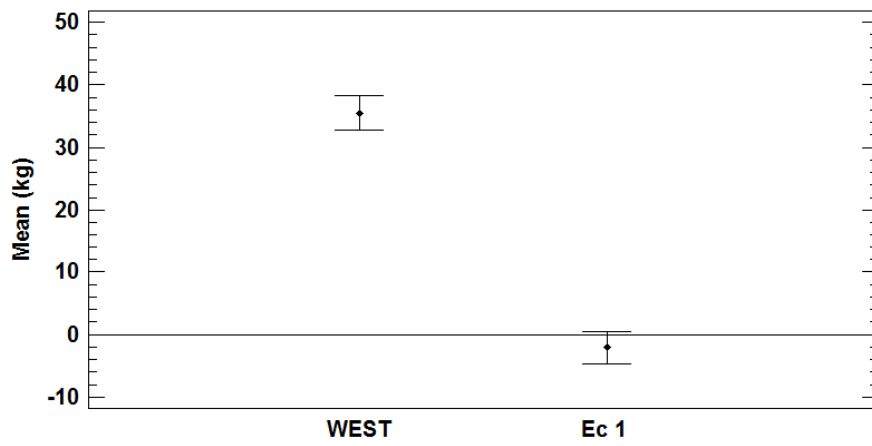


Figure 4A. Means and 95% confidence intervals for the residuals.

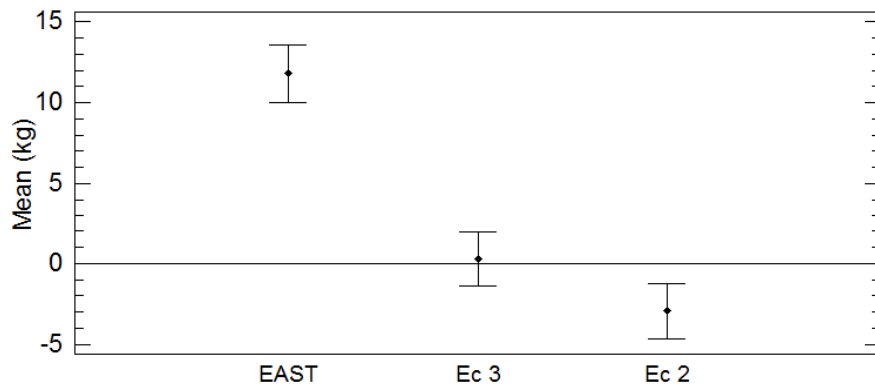
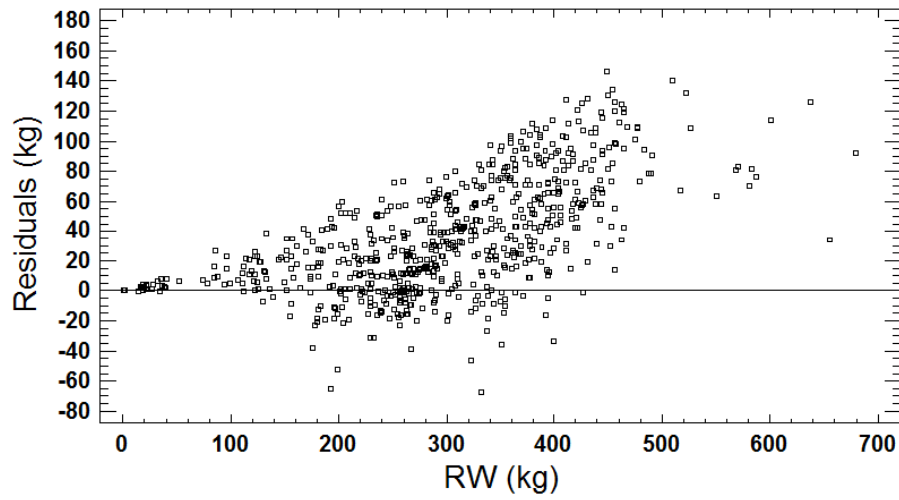


Figure 4B. Means and 95% confidence intervals for the residuals.

A)



B)

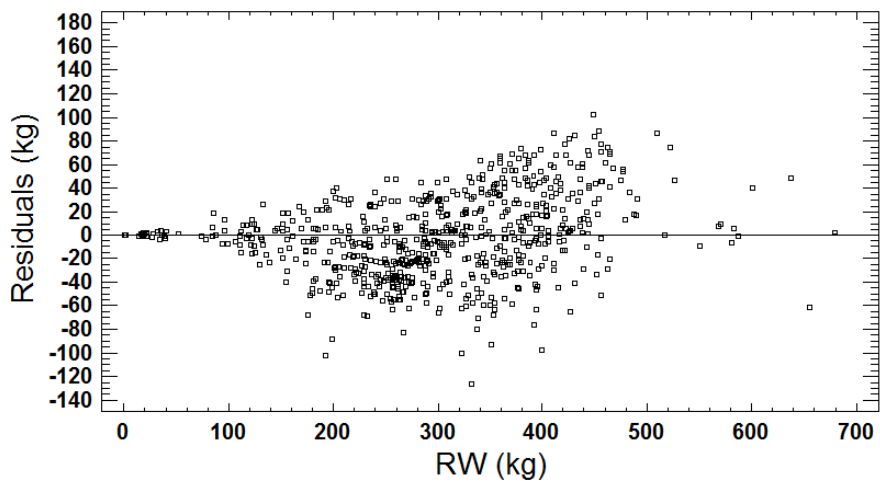
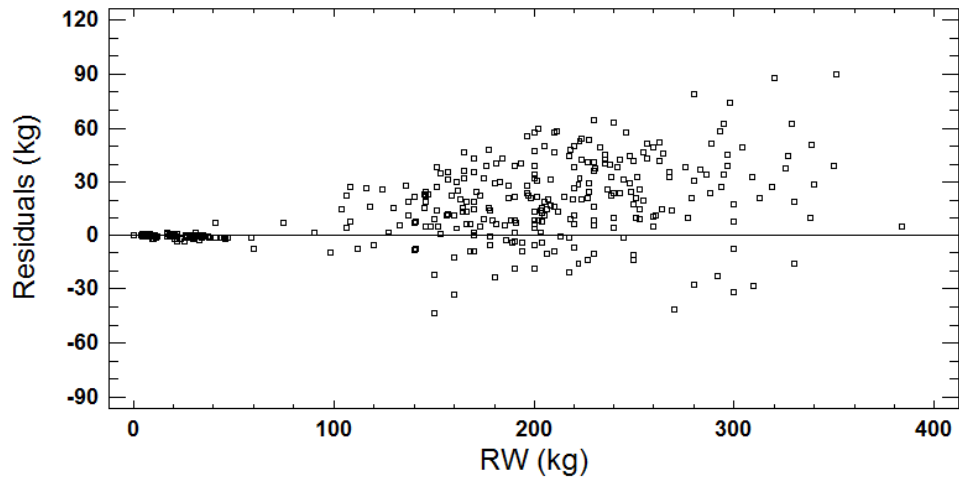
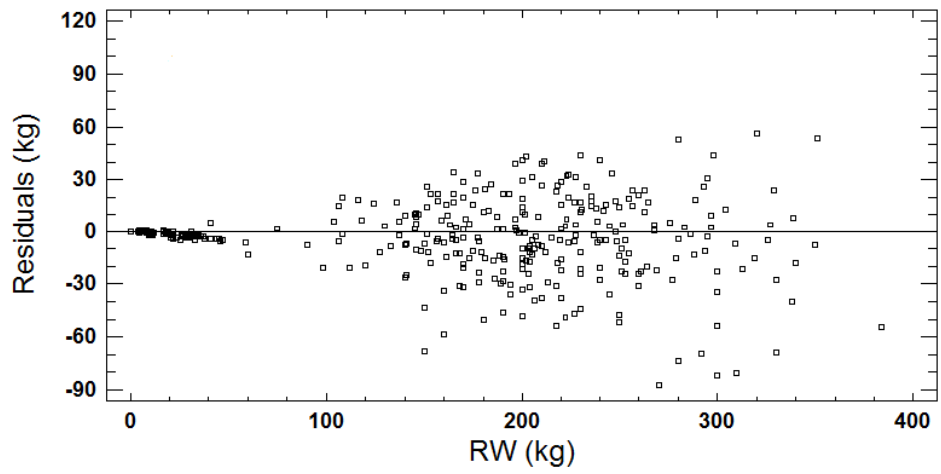


Figure 5. Residual plots. The figures A) and B) correspond to the model linked to the equations *WEST* and *Ec 1*, respectively.

A)



B)



C)

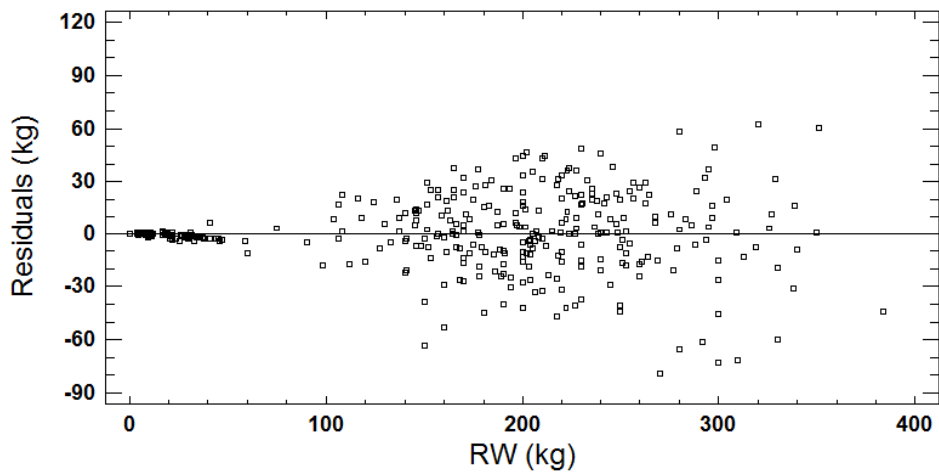


Figure 6. Residual plots. The figures A), B) and C) correspond to the model linked to the equations *EAST*, *Ec 2* and *Ec 3*, respectively.

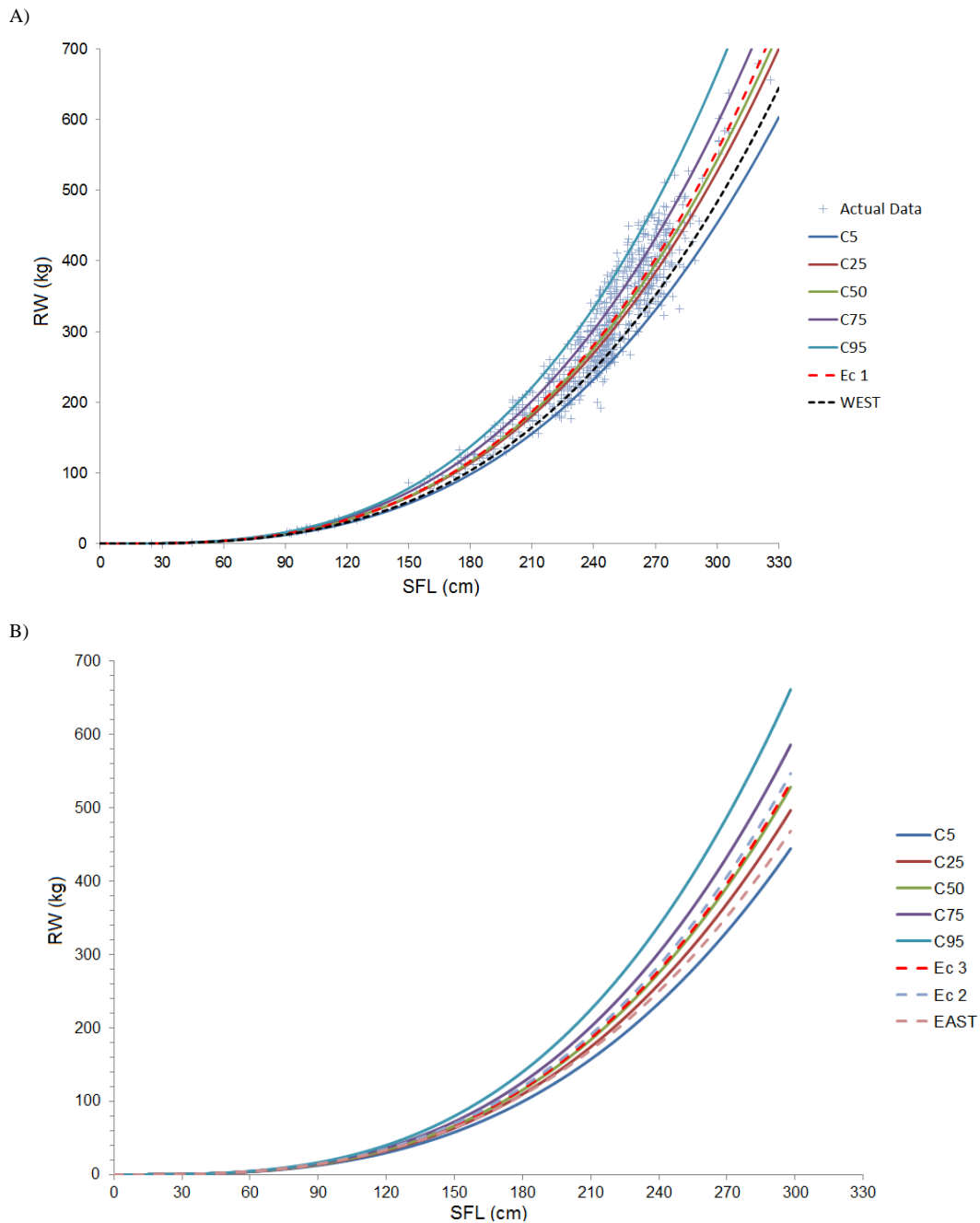


Figure 7.

- A) Graphs corresponding to the selected quantile curves (5%, 25%, 50%, 75% and 95%; solid lines) and to the analyzed models *WEST* and *Ec 1* (dashed lines).
- B) Graphs corresponding to the selected quantile curves (5%, 25%, 50%, 75% and 95%; solid lines) and to the analyzed models *EAST*, *Ec 2* and *Ec 3* (dashed lines).

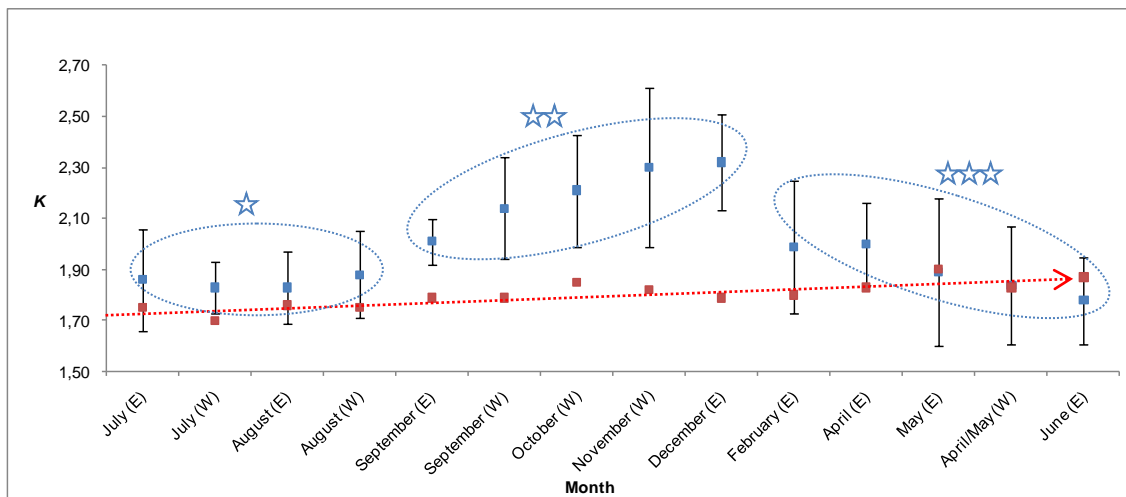
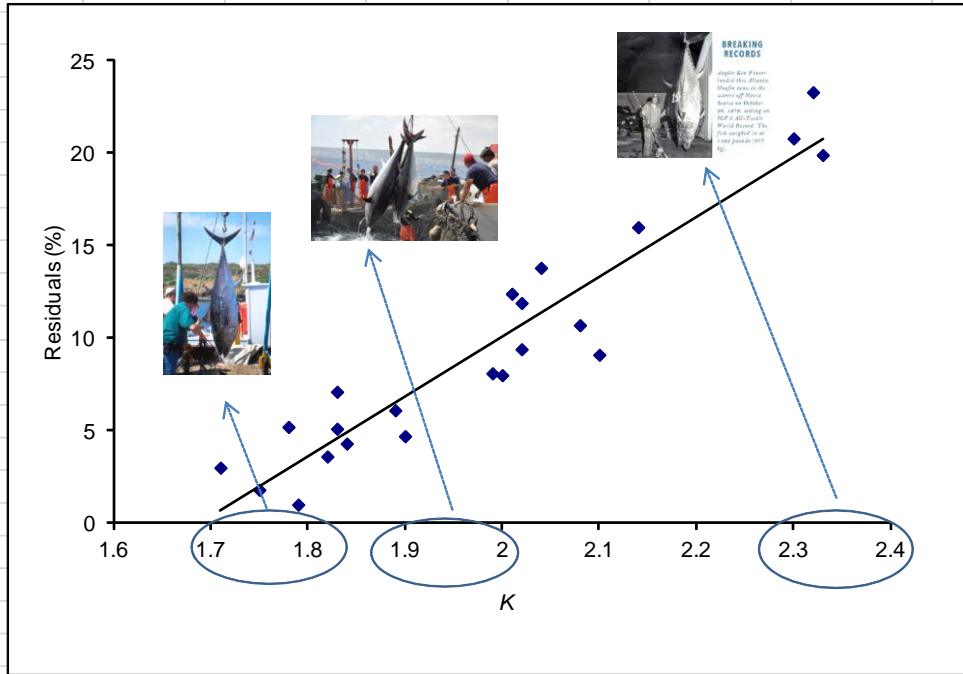


Figure 8. Evolution of K throughout the year.

- Mean values obtained by using data of Tables 10A and 10B
- Values obtained by using monthly L - W adopted equations (Rodríguez-Marín et al., 2015). See data, Table 7
- ⋯→ Trend of K values using monthly L - W adopted equations (Rodríguez-Marín et al., 2015)
- ★ Skinny fish after spawning
- ★★ Fish getting fatter. High trophic phase
- ★★★ Fish slimming during reproduction

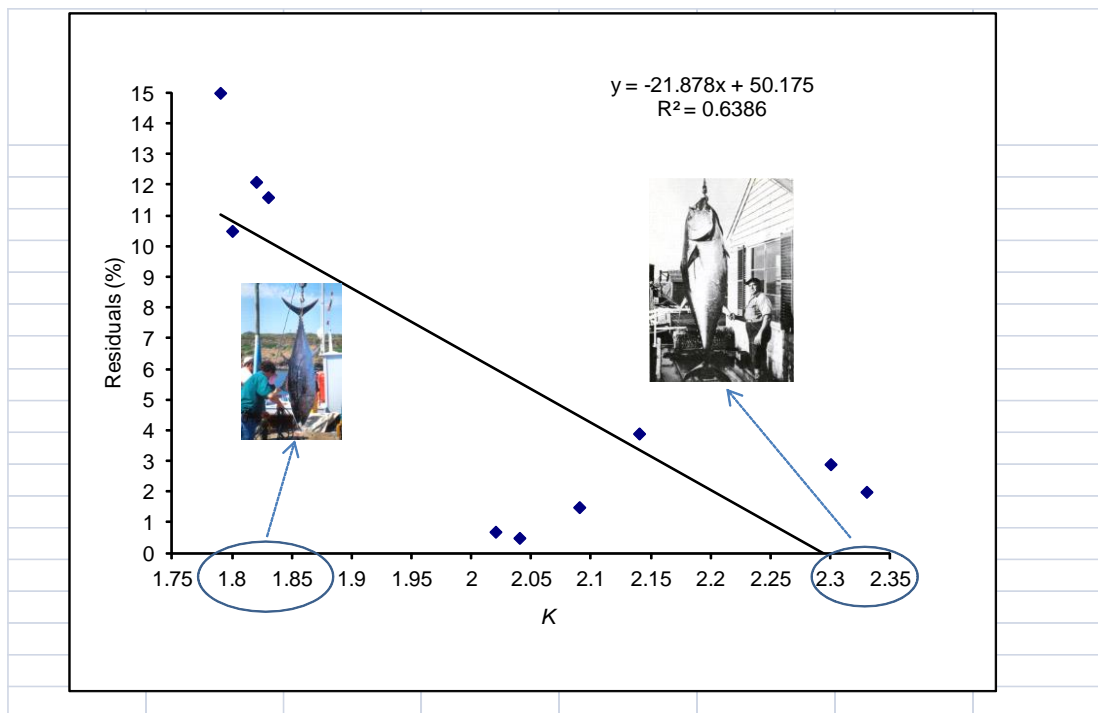
A)



$$y = 32.371x - 54.66$$

$$R^2 = 0.9008$$

B)



C

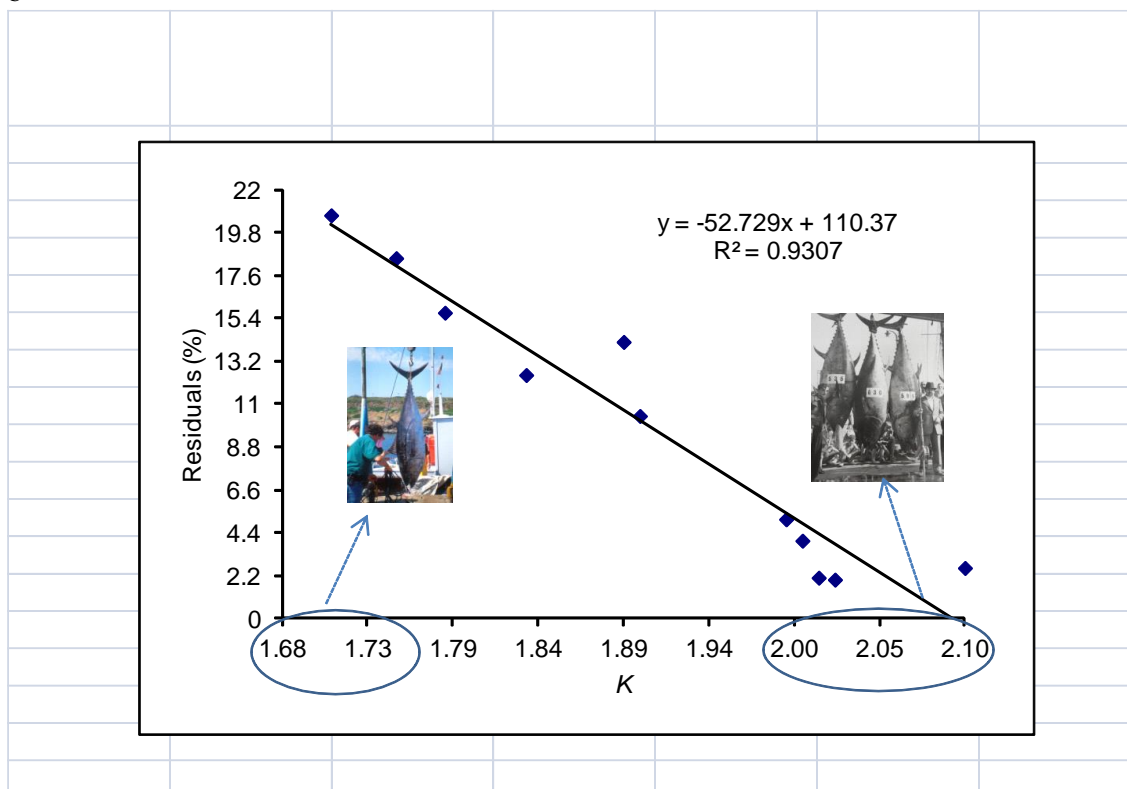


Figure 9.

- Relationship between the value of K (samples in **Tables 10A** and **10B**) and the residual obtained by applying the monthly length-weight adopted equations published by Rodríguez-Marín et al. (2015)
- Relationship between the value of K (samples in **Table 10A**) and the residual obtained by applying the discarded equation of Parrack and Phares (1979) for the western stock.
- Relationship between the value of K (samples in **Table 10B**) and the residual obtained by applying the discarded equation of Arena (in ICCAT, 2010) for the eastern stock.

PHOTO GALLERY

$K \leq 1.7$ (skinny ABFT), generally after spawning (July-August)

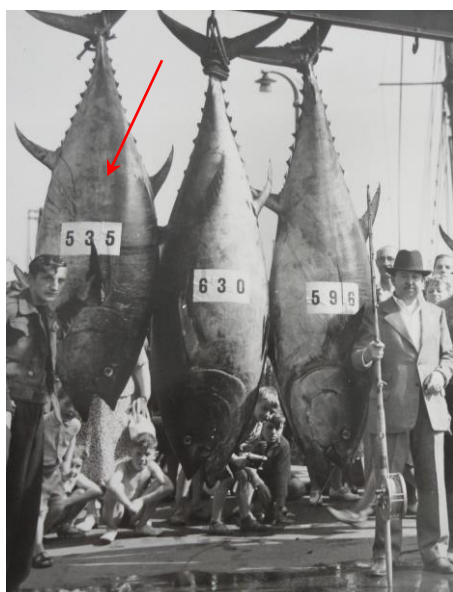


Sardinia (Italy)
SFL \approx 195 cm; *W* = 115 kg
K = 1.5

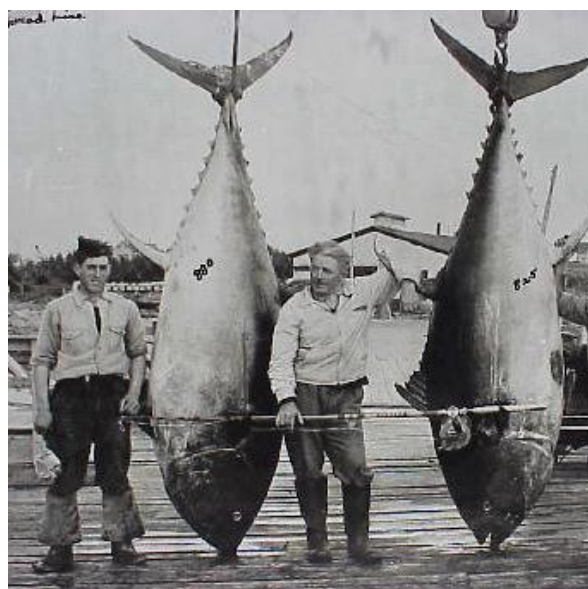


Javea (Spain)
SFL \approx 215 cm; *W* = 140 kg
K \approx 1.4

Unusual shape (→) of a very fat ABFT in high trophic phase (September-December)



Scarborough (UK), September-1948
SFL \approx 223 cm
W = 243 kg (535 Lb)
K \approx 2.2



New Scotia (Canada)
SFL \approx 255 cm
W \approx 400 kg
K \approx 2.2

$K > 2.2$ (very fat ABFT), generally during high trophic phase (September-December)

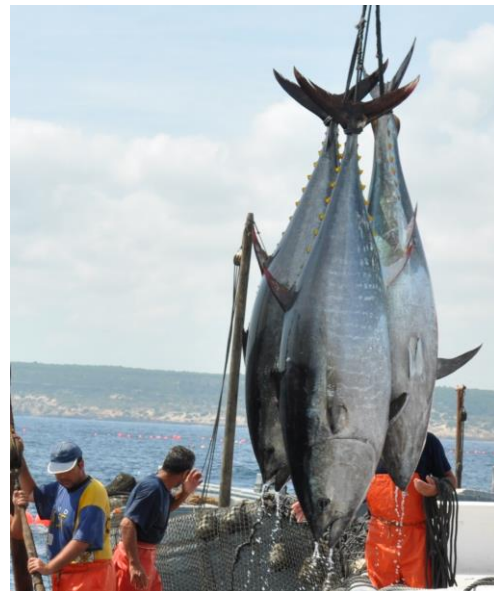


Massachusetts (USA), October-1950
(Mather, 1963)
 $SFL = 270$ cm; $W = 428$ kg
 $K = 2.2$



New Scotia (Canada), October-1979
(Current Guinness World Record)
(Fraser, 2008)
 $SFL = 304-320$ cm; $W = 679$ kg
 $K = 2.4-2.1$

$K \geq 1.8-2.0$, generally before spawning (April-May)



Barbate trap (Cádiz coast, Spain)

COMPARISON OF THE MORPHOLOGY

A skinny ABFT in low fattening condition ($K \leq 1.7$, left);
a very fat in high fattening condition ($K \geq 2.2$, right)

