

## A NOTE ON THE SELECTION OF STOCK-RECRUITMENT RELATIONSHIPS FOR WESTERN ATLANTIC BLUEFIN TUNA, WITH REFERENCE TO TEMPORAL VARIATION IN THE RELATIONSHIP

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### SUMMARY

*The fitting of stock-recruitment relationships (SRRs) for western Atlantic bluefin tuna is re-examined with respect to the choice of model (Beverton-Holt vs 2-line model) and the invocation of a "regime change" in 1976 to explain lower annual recruitments estimated in subsequent years. The Beverton-Holt and 2-line models are expressed in terms of the parameter  $a$  (steepness) and  $K$  (a reference biomass level, equal to the "pivot point" in the 2-line model). Allowing either  $a$  or  $K$  to vary as a stationary Gaussian process results in better fits to the recruitment estimates than assuming a one-off change in 1976. The variable  $K$  model provides the best fit. This model is used to define the management reference points  $F_{MSY}$  and  $B_{MSY}$  in a way that remains valid with a time-varying SRR. This way of modelling temporal variation in the SRR has the following advantages: (i) it is not necessary to assume a sudden change at any point in the series or to arbitrarily discard part of the series; (ii) the possibility of continuing or future changes in the SRR is allowed for.*

### RÉSUMÉ

*L'ajustement des relations stock-recrutement (SRR) pour le thon rouge de l'Atlantique Ouest a été réexaminé en ce qui concerne le choix du modèle (modèle de Beverton-Holt par opposition à modèle à 2 lignes) et l'invocation d'un « changement de régime » en 1976 pour expliquer les recrutements annuels plus faibles estimés au cours des années suivantes. Les modèles Beverton - Holt et à deux lignes sont exprimés en termes du paramètre  $a$  (pente à l'origine de la relation stock-recrutement « steepness ») et  $K$  (un niveau de biomasse de référence égal au « point de pivot » dans le modèle à deux lignes). Permettre à  $a$  ou  $K$  de varier en tant que processus gaussien stationnaire se traduit par de meilleurs ajustements des estimations du recrutement par rapport au postulat d'un changement unique en 1976. Le modèle variable- $K$  fournit le meilleur ajustement. Ce modèle est utilisé pour définir les points de référence de gestion  $F_{PME}$  et  $B_{PME}$  d'une manière qui reste valable avec une SRR variant dans le temps. Cette façon de modéliser la variation temporelle de la SRR présente les avantages suivants : (i) il n'est pas nécessaire de postuler un changement soudain à un moment de la série ou de rejeter arbitrairement une partie de la série et (ii) la possibilité de poursuite ou de changements futurs de la SRR est prise en compte.*

### RESUMEN

*Se reexamina el ajuste de las relaciones stock-reclutamiento (SRR) para el stock de atún rojo del Atlántico oeste respecto a la elección del modelo (Beverton-Holt frente al modelo de 2 líneas) y la invocación de un "cambio de régimen" en 1976 para explicar los menores reclutamientos anuales estimados en años posteriores. Los modelos de Beverton y Holt y de dos líneas se expresan en términos del parámetro  $a$  (inclinación) y el parámetro  $K$  (un nivel de biomasa de referencia igual al punto de inflexión en el modelo de dos líneas). Permitir a " $a$ " o a " $K$ " variar como un proceso estacional gaussiano produce ajustes mejores a las estimaciones de reclutamiento que el supuesto de un solo cambio en 1976. El modelo de variable  $K$  proporciona el mejor ajuste. Este modelo se utiliza para definir los puntos de referencia de ordenación  $F_{RMS}$  y  $B_{RMS}$  de tal modo que sigan siendo válidos con una relación stock-reclutamiento (SRR) que varía en el tiempo. Esta manera de modelar la variación temporal en la SRR tiene las siguientes ventajas: (i) no es necesario asumir un cambio súbito en cualquier punto de la serie o descartar arbitrariamente una parte de la serie; (ii) brinda la posibilidad de considerar la continuación de la SSR o de futuros cambios en la SSR.*

### KEYWORDS

*Stock assessment, recruitment, stochastic processes, potential yield, fishing mortality*

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## 1. Introduction

The choice of stock-recruitment model for use in assessing the current status of and projecting the recovery of the western Atlantic bluefin stock has been discussed on several occasions in the ICCAT SCRS over the years. During the 2014 stock assessment meeting, it was agreed to fit two stock-recruitment models to the times series of recruitment and spawning stock biomass resulting from the VPA stock assessment of the western Atlantic bluefin tuna stock: (i) the Beverton-Holt model; and (ii) a “two-line” model that that discarded data prior to 1976 on the grounds that there could have been a discontinuous shift to a permanent, lower-recruitment regime from 1976 onwards (ICCAT, 2015).

An analysis performed during the SCRS plenary (Anon., 2015a) suggested that the “three-line” model, being the two-line model supplemented by a discontinuous change in 1976, provided significantly better fits to the time series of stock and recruitment estimates arising from the 2012 and 2014 SCRS assessments than did the Beverton-Holt model fit to the entire series of stock and recruitment estimates.

This note re-examines the question of the stock-recruitment relationship for western Atlantic bluefin tuna with a view to finding a way to allow for possible temporal variation in the stock-recruitment relationships over and above the usual annual fluctuations.

## 2. Material and methods

### 2.1 Stock recruitment models

The “two-line” stock-recruitment relationship traditionally used by the SCRS for BFT assessments assumes that the stock-recruitment relationship is linear from the origin up to a pivot point, and flat for spawning stock levels above that point. The Beverton-Holt (BH) stock-recruitment model predicts that mean recruitment increases with increasing biomass towards some asymptotic level (**Figure 1**). In the 2012 and 2014 SCRS assessments of the western Atlantic Bluefin stock (BFT-W), the two-line model was fit to the stock and recruitment estimates for the period 1976 onwards only, omitting the pre-1976 estimates. The Beverton-Holt model was fitted to the entire time series from 1970. The “three-line” model, fitted in Anon., 2015a to the result of the 2012 and 2014 SCRS assessments for BFT-W, was an extension of the two-line model that allows for a one-off change in the stock-recruitment relationship from 1976.

The Beverton-Holt stock-recruitment model can be given by:

$$R_{t+1} = \frac{aB_t}{1 + B_t/K} e^{\sigma v_t} \quad (1)$$

where  $R_{t+1}$  is the number of 1-year old recruits in year  $t+1$ ,  $B_t$  is the spawning stock biomass in year  $t$ , and  $a$  and  $K$  are parameters to estimate.  $a$  is a measure of the steepness of the relationship.  $K$  is related to, but not coincident with, the theoretical unfishable biomass level. The noise terms  $v_t$  are random  $N(0,1)$  normal deviates associated with the year-to-year fluctuations in recruitment.  $\sigma$  is a parameter determining the level of variability.

The two-line stock-recruitment relationship is given by:

$$R_{t+1} = \begin{cases} aB_t e^{\sigma v_t} & (B_t \leq K) \\ aK e^{\sigma v_t} & (B_t \geq K) \end{cases} \quad (2)$$

For both the BH and 2-line models, the median recruitment is equal to or approaches  $aK$  at high stock sizes (the asymptote). The likelihood profile for  $K$  in the two-line model is by definition flat both below and above the range of SSB levels used to fit the model. The BH model yields a nowhere-flat likelihood, but when fitting either the 2-line or the BH model, three kinds of results are possible: (i) there is a best estimate of  $K$  within the observed range of spawning stock sizes; (ii) there is no best estimate of  $K$ , but values of  $K$  below the observed range of stock sizes are favored (have higher likelihood); (iii) there is no best estimate of  $K$ , but values above the observed range of stock sizes are favored.

In case (ii), a point estimate is obtained of the asymptotic recruitment level  $aK$  but not of the slope  $a$ . In case (iii), a point estimate is obtained of the slope  $a$  but not of the asymptotic recruitment level  $aK$ .

## 2.2 Changes in parameters over time

Anon., 2015a considered a possible discontinuous change in 1976 in the context of the two-line model but not with the Beverton-Holt model. There was no reason given not to use both models with both hypotheses. Also, it is unclear which parameter ( $a$  or  $K$ ) should have changed in 1976. Therefore, both the 2-line and the BH models are refit here under the following hypotheses: (i) constant parameters; (ii)  $a$  changes in 1976; (iii)  $K$  changes in 1976.

An hypothesized one-off historical change (“regime change”) does not provide a complete model of the variation in stock-recruitment relationship until it is also specified what further such changes might be expected, in terms of their nature and of their frequency. Once the possibility of a change in the stock-recruitment relationship has been accepted, it is not reasonable to rule out the possibility of further changes. The possibility of further changes can be modeled by supposing that some or all of the parameters of the stock-recruitment relationship are subject to stochastic processes that yield multi-year changes in parameter values over and above the conventionally assumed annual variability. The simplest such process is a Gaussian process,  $\varphi(t)$  where the covariance between the residuals at two times  $t$  and  $t'$  is given by  $\text{cov}(\varphi_t, \varphi_{t'}) = \alpha \exp(-\beta(t-t')^2)$ .

The process is stationary, in the sense that its behavior does not change over the longer term.

The fitting of the Gaussian process is achieved indirectly, via the fitting of a Fourier series with stochastic coefficients. Because the fitting method only works with differentiable functions, the Gaussian process was fitted in conjunction with the BH model only. Two alternative models involving a Gaussian process were considered: (i)  $\log a$  varies as a Gaussian process; and (ii)  $\log K$  varies as a Gaussian process. The logarithms were taken to ensure that the parameters  $a$  and  $K$  remain positive.

The fits of the various models are compared using the AICc criterion. Following Anon., 2015a, one parameter was added to the parameter count for the regime-change models to account for the fact that the position of the change-point (1975/6) was selected from the data. The effective number of parameters for Gaussian process models is typically fractional.

## 3. Results

**Table 1** gives the parameter estimates and goodness of fit (as measured by the AICc criterion) for the BH and 2-line models to the time series of stock and recruitment from the 2014 SCRS assessment of western Atlantic BFT. When fitting to the full series (1970-2010) without parameter changes, the BH model gives a better fit than the 2-line model ( $\Delta\text{AICc} = 5.4$ ). The 2-line model provides no point estimate of  $K$ : the best fit is obtained by a line through the origin without a pivot point in the observed range of SSBs.

When allowing a change in 1976, both the 2-line and BH models result in estimates of  $K$  that are below the observed range of SSB's, that is: flat stock-recruitment relationships are estimated for each model, regardless of which parameter is allowed to have changed in 1976. Thus, the fits that result from allowing a change in  $K$  or a change in  $a$  are identical. Allowing for parameter change in 1976 substantially reduced the AICc for the fits of both the 2-line and BH models.

The Gaussian process models yield slightly better (lower) AICc's than the regime-change model. This implies that there is no need to invoke non-stationary changes to explain the observed pattern of stock and recruitment. Of the two Gaussian process models, the one involving variation in  $K$  yielded a slightly lower AICc. Variability in  $K$  results in a proportionally greater variability in recruitment at higher stock sizes than at lower stock sizes.

**Figure 2** shows the estimated recruitments and SSB values from the 2014 assessment, along with the fitted values from the 3-line model and the Gaussian process for  $K$ . The results show that the Gaussian process model, which does not assume a 1-off change in a particular year, can provide a reasonable fit to the series of stock and recruitment estimates.

#### 4. Implications for management reference points

The possibility of variation in the stock-recruitment relationship has implications for management reference points. These are typically defined for the case of constant parameters and may need to be adapted to apply to cases with variable parameters.

In the constant parameter case, the reference points  $F_{MSY}$  and  $B_{MSY}$  are conventionally calculated as follows. The spawning-biomass-per-recruit is calculated as:

$$SSBR = \sum_{a=1}^{\infty} \exp\left(-\sum_{j=1}^{a-1} Z_j\right) w_a p_a \quad (3)$$

And the yield-per-recruit is given by:

$$YPR = \sum_{a=1}^{\infty} \exp\left(-\sum_{j=1}^{a-1} Z_j\right) w_a \frac{F_a}{Z_a} \quad (4)$$

where  $Z_j = M_j + F_j$  is the total (natural + fishing) mortality rate for age class  $j$ ;  $w_a$  is the mean weight of fish aged  $a$ ; and  $p_a$  is the proportion of aged  $a$  fish that participate in spawning. The equilibrium biomass for a given set of age-specific fishing mortalities,  $\{F_a\}$  is that biomass level where the SSBR from equation (3) matches the biomass per recruit implied by equations (1) or (2). This equilibrium biomass can be denoted  $B_{eqm}(\{F_a\})$ . If  $F_a$  is given by  $F \cdot S_a$  where  $\{S_a\}$  is a fixed vector of age-specific selectivities, then  $B_{eqm}$  is a function of  $F$ . The fishing mortality reference point  $F_{MSY}$  is the value of  $F$  which maximizes the total yield. The total yield is the product of the yield-per-recruit (4) and the recruitment level corresponding to  $B_{eqm}(F)$ . The biomass reference point  $B_{MSY}$  corresponds to  $B_{eqm}(F_{MSY})$ . Its value depends on the age-specific selectivities  $\{S_a\}$ .

The parameter  $K$  serves as a simple scaling factor for the stock dynamics model. Reference points expressed in terms of fishing mortality, such as  $F_{MSY}$  or  $F_{0.1}$ , are independent of  $K$  and therefore formally unaffected by variations in  $K$ . This is because the equilibrium biomass and equilibrium yield that correspond to any given value of  $F$  are each proportional to  $K$ . Changing  $K$  changes the height of the yield vs  $F$  curve but not its shape nor the location of its maximum. Biomass-related reference points such as  $B_{MSY}$  are affected by variations in  $K$ : if they are to be used, then they need to be defined in a way that makes sense in the context of a variable  $K$ .

One possible definition of  $B_{MSY}$  is the value of  $B_{MSY}$  that would result from the assumption that all relevant parameters, including  $K$ , would remain constant at their current values. This definition can be denoted the ‘‘pseudo-constant’’  $B_{MSY}$ . An alternative definition proposed by Cooke (2012) is that  $B_{MSY}$  be defined as the biomass level that results from constant- $F$  fishing at  $F = F_{MSY}$ . This definition can be denoted the ‘‘floating’’  $B_{MSY}$ . In both cases,  $B_{MSY}$  is a function of time.

In order to explore the behavior of the different reference points, hypothetical stocks were simulated, with the same growth and mortality parameters as used for the 2014 BFT-W assessment. The stock-recruitment relationship was assumed to be the variable- $K$  Gaussian process fitted in the preceding section (bottom row of Table 1). The stock was assumed to have been, and to continue to be, fished at constant  $F$ , with uniform selectivity from age 1 onwards, and with  $F$  set to  $0.8 \cdot F_{MSY}$ .

The choice of  $F = 0.8 \cdot F_{MSY}$  is arbitrary, but reflects the well-established result that fishing intensities moderately below  $F_{MSY}$  result in yields only slightly less than MSY, while providing higher average stock sizes and lower variability in yield. For the model used here, fishing at  $0.8 \cdot F_{MSY}$  produces an average yield of 97.5% of MSY and an average spawning stock biomass 117% of  $B_{MSY}$ .

**Figure 3** shows some random realizations of the trajectory of spawning stock biomass over a 50-year period from simulations of this model. Also shown are the corresponding trajectories of the two definitions of the  $B_{MSY}$  reference point. The simulations were generated by assuming that the stock was at equilibrium with the given fishing mortality rate at the start of the simulation. The recruitment values in each year were generated randomly as realizations of the stock-recruitment relationship.

Of the two alternative definitions of  $B_{MSY}$  considered here, the ‘‘pseudo-constant’’  $B_{MSY}$  exhibits the greater variability, even though it is defined on the assumption that the current parameters will remain constant. The results in **Figure 3** show that even when the fishing mortality is held below  $F_{MSY}$  throughout, the spawning stock biomass often dips below the pseudo-constant  $B_{MSY}$ . This is because the stock-recruitment parameter values can change quite rapidly from one year to the next, hence the ‘‘pseudo-constant’’  $B_{MSY}$ , which is calculated on the assumption that the new parameter values will remain permanently in effect, can also change rapidly. By contrast, the spawning stock can only recover slowly, as changes in the *per capita* recruitment rate take several years to generate substantial changes in the spawning stock size.

In comparison, the “floating”  $B_{MSY}$  varies more sluggishly because it is defined as the actual trajectory of spawning stock biomass that would pertain if  $F$  were held at  $F_{MSY}$ . In this regard it provides a more stable reference point. Provided  $F$  is held below  $F_{MSY}$ , then the floating  $B_{MSY}$  will, by definition, remain below the current spawning stock biomass level. Hence the blue curve (floating  $B_{MSY}$ ) shown in **Figure 3** remains below the brown curve (current SSB level) in all realizations of the model.

## 5. Conclusions

This preliminary analysis of the stock-recruitment relationship for western Atlantic bluefin tuna based on the results of the 2014 SCRS assessment suggests that the relationship has not been constant. However, it is not necessary to assume a discontinuous, one-off change (“regime shift”). The stock-recruitment relationship can be fitted in terms of a smoothly varying Gaussian process. Management reference points, such as  $F_{MSY}$  and  $B_{MSY}$ , traditionally defined only for the constant parameter case, can be adapted to remain applicable in the variable parameter case. In the case of the variable  $K$  model,  $F_{MSY}$  can be used as a fishing mortality reference point. Of the two alternative options considered here for the definition of  $B_{MSY}$ , the “floating”  $B_{MSY}$  appears to be a more suitable extension of the  $B_{MSY}$  concept to the variable-parameter case than is the “pseudo-constant”  $B_{MSY}$ .

An important caveat to these results is that the fitting of a stock-recruitment relationship to the results of the SCRS stock assessment, as if they represented genuine observations, is a statistically doubtful procedure. The estimates of spawning stock biomass and annual recruitments that arose from the 2014 SCRS assessment are treated in this paper, as they also were by Anon. (2015a), as if they were direct observations. The complex error structure of the assessment results, with potential correlations between errors in stock and recruitment estimates in different years, has been ignored. It would be preferable to embed the fitting of the stock-recruitment relationship into the assessment itself, so that the uncertainty in the assessment is correctly handled in the fitting of the stock-recruitment relationship. This can be achieved by incorporating the stock-recruitment model as a penalty function in the fitting procedure used in the assessment.

The use of such a penalty function should be considered for the 2016 bluefin SCRS assessment, both for fitting the assessment model and for making future projections. The use of a time-varying model for the stock-recruitment process, such as the one presented in this paper, would enable the uncertainty surrounding possible changes in the stock-recruitment relationship to be reflected in the assessment of the current stock level relative to reference points such as  $B_{MSY}$ , and in the projected rebuilding times. The procedure used for the 2014 SCRS assessment was unsatisfactory, in that it was assumed that a change in stock-recruitment parameters had occurred at a specific time in the past, but that there had been, and would be, no further such changes.

## Acknowledgements

This work was supported by The Ocean Foundation and the Pew Charitable Trusts.

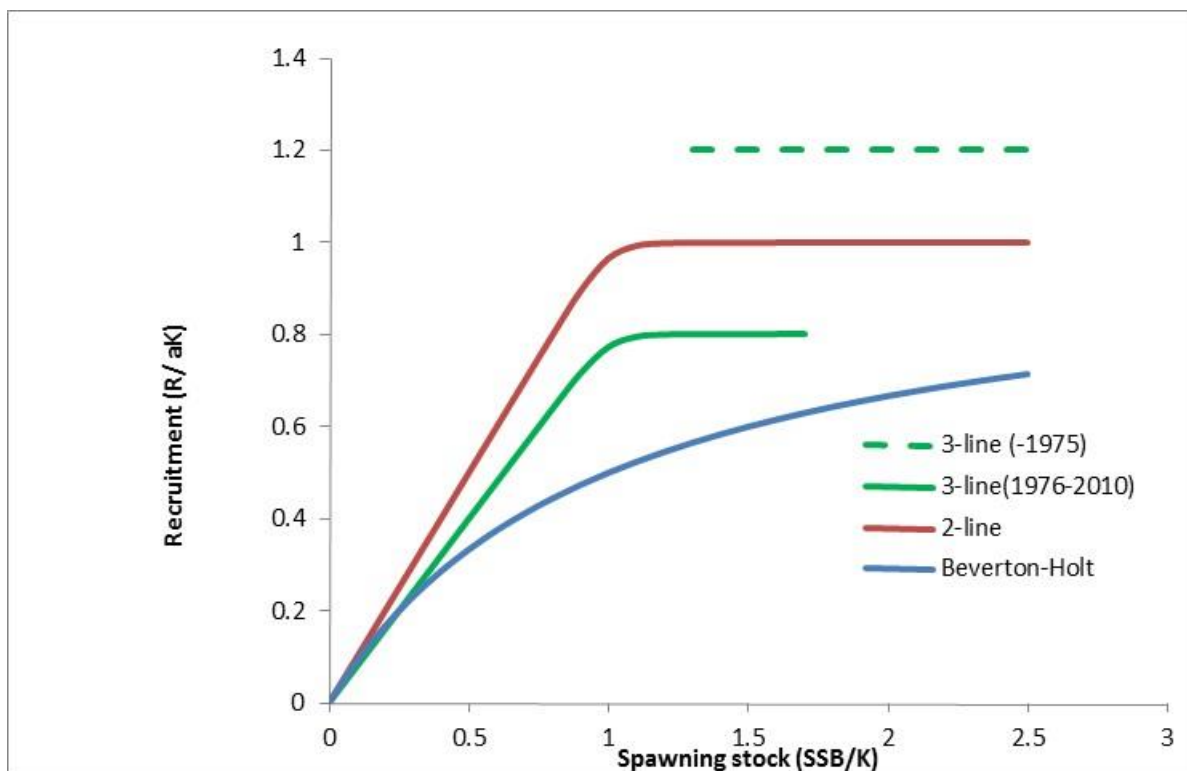
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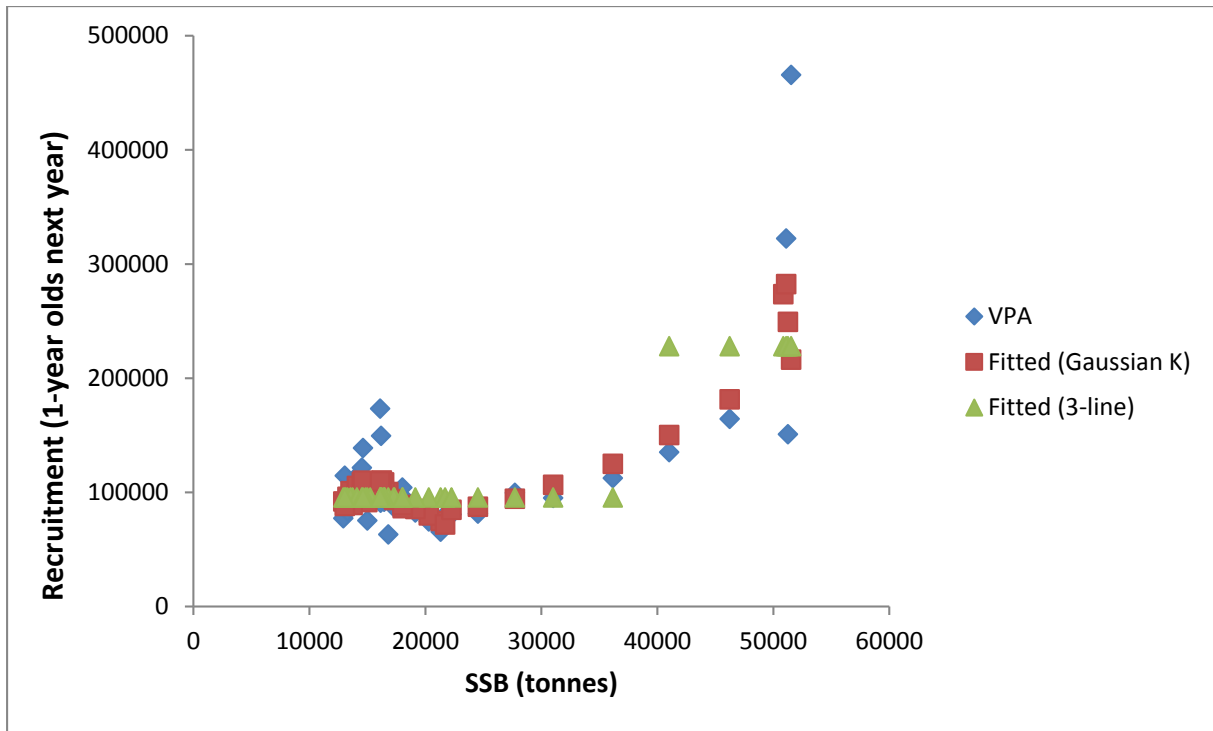
**Table 1.** Results of fitting various models to the stocks and recruitment estimates from the SCRS 2014 BFT-W assessment.

Model	Period	Parameter estimates						AICc
		Slope <i>a</i>	pivot <i>K</i>	asymptote <i>aK</i>	variance $\sigma$	log-likelihood	Effective parameters*	
2-line	1970-2010	5.4	$\infty$	$\infty$	0.36	21.18	2.00	-38.04
	1970-75	$\infty$	0	227 977	0.26	33.72	4.00	-58.30
	1976-2010	$\infty$	0	95 597	0.26	33.72	4.00	-58.30
BH	1970-2010	8.1	42 480	344 689	0.32	25.07	3.00	-43.47
	1970-75	$\infty$	0	227 977	0.26	33.72	4.00	-58.30
	1976-2010	$\infty$	0	95 597	0.26	33.72	4.00	-58.30
BH	variable <i>a</i> 1970-2010	4.8	37 751	182 338	0.22	41.35	8.13	-61.62
BH	variable <i>K</i> 1970-2010	5.3	31 267	166 341	0.22	41.41	7.76	-62.94

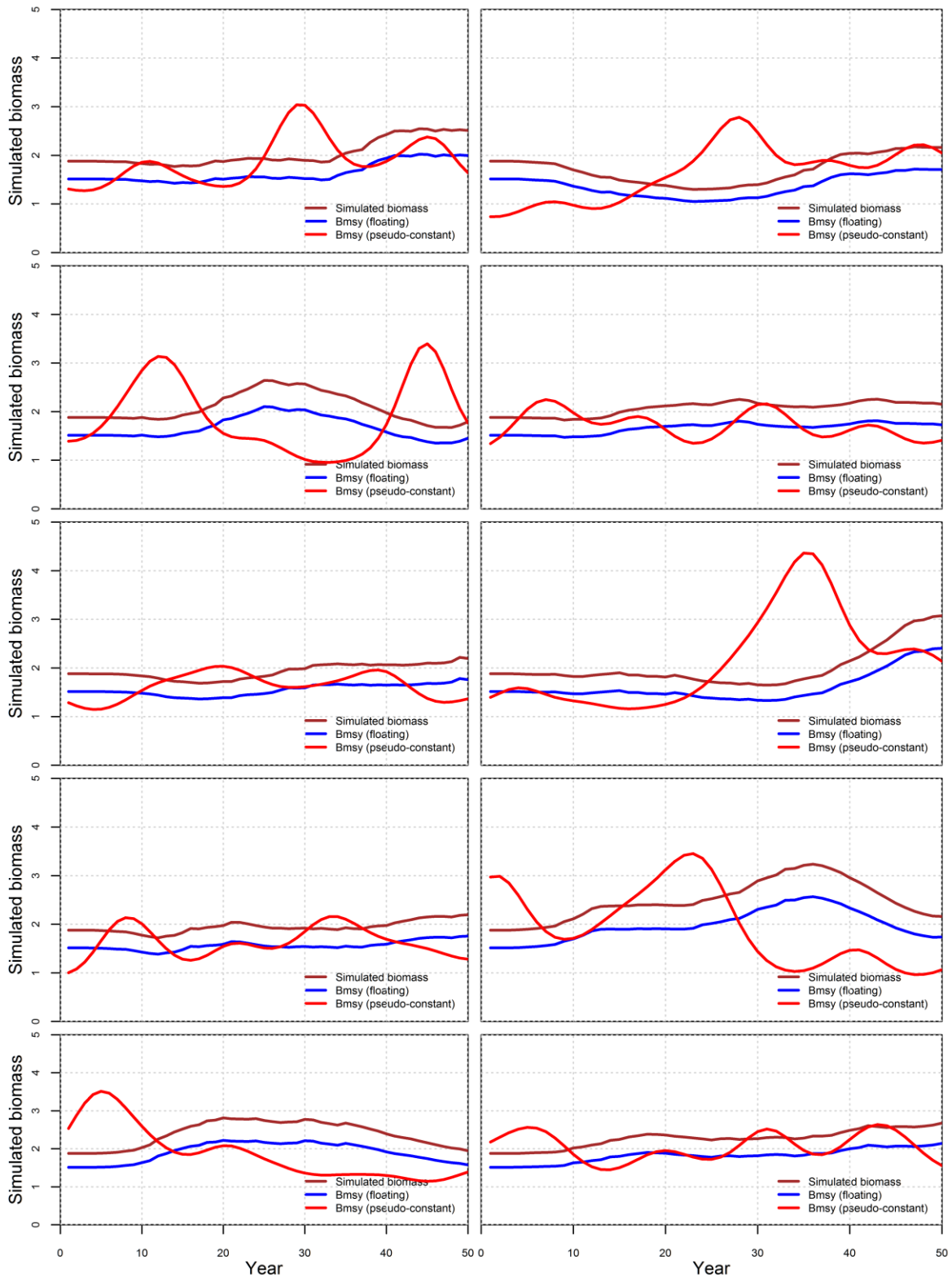
\* excluding degenerate parameters (e.g.  $K \rightarrow 0$  or  $K \rightarrow \infty$ )



**Figure 1.** Beverton-Holt and 2-line stock-recruitment relationships. The 3-line model is the same as the 2-line model but with two different values for the asymptote corresponding to different time periods.



**Figure 2.** Estimated recruitments and SSB values from the 2014 assessment, with fitted values from the 3-line model (green triangles) and the Gaussian process for  $\log K$  (red squares). The Gaussian process model shown is the one specified in the last row of **Table 1**.



**Figure 3.** Examples of stochastic realizations of a stock fished at  $0.8 \cdot F_{MSY}$ . Growth and mortality parameters are those used in the SCRS 2014 BFT-W assessment. Recruitment was generated using the variable- $K$  model as fitted in this paper. The simulated biomass is the spawning stock biomass realized by running the model forward from a stock starting at an assumed equilibrium with the given  $F$  level. The “floating”  $B_{MSY}$  (blue line) is the stock level that would pertain if  $F$  were kept equal to  $F_{MSY}$ . The “pseudo-constant”  $B_{MSY}$  level in any given year is the notional  $B_{MSY}$  that would result from keeping the stock-recruitment parameters constant at the values of the given year. Biomass levels are expressed as multiples of the mean value of  $K$  in the stock-recruitment model.