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SOME THEORETICAL CONSIDERATION ON NONEQUILIBRIUM PRODUCTION MODELS

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SUMMARY

In this paper, a general formulation for the production models in non-equilibrium situation is presented, using a common definition to all models of the parameter r (intrinsic rate of natural growth) and an adopted transformation function of the biomass. A definition of mean biomass is presented through the transformation function and the mathematical formulation of the models (Schaefer, Fox and Pella & Tomlinson forms) are derived for mean biomass, annual CPUE and annual fishing effort. An expression for obtaining the initial annual biomass from the mean biomass is derived. Approximate expressions, useful in fitting procedures of the models to the data, are also presented.

RESUMÉ

Le présent document présente une formule générale pour les modèles de production en conditions de non-équilibre, en utilisant une définition commune à tous les modèles du paramètre r (taux intrinsèque de croissance naturelle) et une fonction de transformation adaptée de la biomasse. On présente une définition de la biomasse moyenne par la fonction de transformation, et la formule mathématique des modèles (formules de Schaefer, Fox et Pella & Tomlinson) est calculée pour la biomasse moyenne, la CPUE annuelle et l'effort de pêche annuel. On calcule une formule pour obtenir la biomasse annuelle initiale à partir de la biomasse moyenne. On présente aussi des formules approximatives, qui sont utiles pour les processus d'ajustement des modèles aux données.

RESUMEN

En este documento se presenta una formulación general para los modelos de producción para situación de no equilibrio, utilizando una definción común a todos los modelos del parámetro r (proporción intrínseca de crecimiento natural) y una función de transformación adoptada de la biomasa. Se presenta una definición de biomasa media mediante la función de transformación y la formulación matemática de los modelos (fórmulas de Schaefer, Fox y Pella & Tomlinson) se deriva para la biomasa media, CPUE anual y esfuerzo de pesca anual. Se deriva una expresión para obtener la biomasa anual inicial a partir de la biomasa media. También se presentan expresiones aproximadas, útiles para ajustar los procedimientos de los modelos a los datos.

Introduction

In last years, Production Models in non-equilibrium situations, have been developed, not only, in biological research but also in economical analyses (Walter & Hilborn 1976, Schnute 1977, Butterworth & Andrew 1984, Clarke *et al* 1992, Yoshimoto *et al* 1993, Prager 1994).

Fox model (Fox 1970, 1975) Schaefer model (Schaefer 1954, 1957) and also the more general GENPROD model (Pella & Tomlinson 1969) have been applied to several fisheries, particularly in the assessment of tuna made by ICCAT and others agencies, for obtaining short term projections of yields and cpue's.

The fishery research papers on this matter are in recent years preferring the parameters r (the intrinsic rate of growth) and k (the carrying capacity),to the parameters k (rate of growth) and Bo (sometimes B_∞) designated by "virgin" biomass or asymptotically natural biomass. Those parameters were used in initial studies and are very common in ecological and economical studies (Anderson 1977, Clarke 1980, Kingsland 1982). In this paper it will be followed the recent trend of using the r and k parameters, that is, the intrinsic rate of growth and the carrying capacity, respectively.

One has to say that the definition of the parameter r is difficulty to find in the literature except for the case of the Schaefer model. This definition is not applicable to the Fox model. In order to get a definition common to all the models, a new definition of mean biomass is presented through the transformation function. The transformation is not a new one. In fishery studies is known as the GENPROD transformation and it is very common in statistics and in models theory.

The mathematical formulation of this models, in recent fishery literature deals with the projections of the initial annual biomasses instead of the mean biomass during each year (Butterworth and Andrew 1984, Hilborn & Walters 1992, Polacheck *et al* 1993, Prager 1994). Both projections are important for the fishery biologists, but the basic data (catches, cpue's and fishing efforts) to fit the models, are associated with the years and not with the values at beginning of the years. For this reason, in this document the models are derived for mean biomasses and also for annual cpue's and annual fishing efforts. An expression is presented for obtaining the initial annual biomasses from the mean biomasses.

A third problem relates to the definition of the mean biomass. The mathematical formulation of production models used in fishery studies, deals with a transformation function of the biomass and not directly with the biomass. The mean biomass, only in very restricted cases, is easily obtained from the function adopted. One of this easy cases occurs when the transformed function is the linear function (Schaefer model). For this reason, in this document, a different definition of the mean biomass during the year is used instead of the mean value of a continuous function during a certain interval.

The fit of the models to the data is done using the common procedures, for instance, the least square method with the Gauss-Newton or simplex techniques. The application of most of this methods, starts from the adoption of some initial values of the parameters to be estimated. For this reason, can be useful to have approximate methods, which produce estimates to be used as initial values in more sophisticated techniques. Approximate expressions are then presented in the paper.

1 - Basic assumptions of the production models

The derivative of a function will be designated by "tia" (instantaneous absolute rate of change) and the derivative of the ln(function) (instantaneous relative rate of change) will be designated by "tir". Then $tir(B_t)$ represents the derivative of $Ln(B_t)$ or $tia(B_t)/B_t$.

The basic assumption of the production models used in fishery research can be formulated as:

$$\underset{TOTAL}{tir\ (B_t)} = \underset{NATURAL}{tir\ (B_t)} + \underset{FISHING}{tir\ (B_t)} \qquad (1.1)$$

Let assume that

$$\underset{FISHING}{tir\ (B_t)} = -F_i = \text{constant in the time interval } T_i \text{ or } (t_i, t_{i+1}).$$

The equation (1.1) will become:

$$\underset{TOTAL}{tir(B_t)} = \underset{NATURAL}{tir(B_t)} - F_i \qquad (1.2)$$

2 - Transformation function, H (B_t)

In order to get a common definition of the intrinsic rate growth to the different models, it is convenient to use a transformation function $H(B_t)$, defined as:

$$H(B_t) = \frac{1-\left(\frac{B_t}{K}\right)^{-p}}{-p} \qquad (2.1)$$

where p is a parameter ($p \geq 0$)
B_t = Biomass at instant t
K = carrying capacity

From equation (2.1) one can obtain the two useful expressions:

$$\left(\frac{B_t}{K}\right)^{-p} = 1 + p.H(B_t) \qquad (2.2)$$

and

$$B_t = K.\left[1 + p.H(B_t)\right]^{-\frac{1}{p}} \qquad (2.3)$$

These expressions as well as the following relationships between tir $[H(B_t)]$ and tir(B_t) will be used later in this paper:

$$\text{tir}[H(B_t)] = -\frac{1+p.H(B_t)}{H(B_t)}.\text{tir}(B_t) \qquad (2.4)$$

and

$$\text{tir}(B_t) = -\frac{H(B_t)}{1+p.H(B_t)}.\text{tir}[H(B_t)] \qquad (2.5)$$

3 - Basic assumption on the NATURAL evolution of B_t

$$\underset{\text{NATURAL}}{\text{tir}\ [H(B_t)]} = -r = \text{constant} \qquad (3.1)$$

with: r = intrinsic rate of NATURAL growth of B_t

This definition is common for the different models

3.1 Particular cases

The following table presents some results for the production models most used in fishery studies:

MODEL	p	$H(B_t)$	tir(B_t)
Schaefer	1	$\left(\frac{K}{B_t}\right)-1$	$r.\left(1-\frac{B_t}{K}\right)$
Fox	0	$Ln\left(\frac{K}{B_t}\right)$	$r.Ln\left(\frac{K}{B_t}\right)$
GENPROD	p>0	$\frac{\left(\frac{K}{B_t}\right)^p-1}{p}$	$r.\frac{1-\left(\frac{B_t}{K}\right)^p}{p}$

The designation of GENPROD will be reserved, in a non corrected but useful way, for p>0.

4. Basic assumption in terms of $H(B_t)$

From eq. (2.5) and eq. (3.1) one can rewrite expression (1.2) as:

$$-\frac{H(B_t)}{1+p.H(B_t)}.\text{tir}[H(B_t)] = r.\frac{H(B_t)}{1+p.H(B_t)} - F_i$$

or,

$H(B_t).\text{tir}[H(B_t)] = -H(B_t)+F_i+p.F_i.H(B_t)$

as $H(B_t).\text{tir}[H(B_t)] = \text{tia}[H(B_t)]$, it will be:

$$\text{tia}[H(B_t)] = F_i-(r-p.F_i).H(B_t) \qquad (4.1)$$

Using the expression $F_i-(r-p.F_i).H(B_t)$ and considering that

tir $(F_i-(r-p.F_i).H(B_t)) = \text{tia}\ \{\ln[F_i-(r-p.F_i).H(B_t)]\}$

one can write:

$$\text{tir}[F_i-(r-p.F_i).H(B_t)] = -(r-p.F_i).\frac{\text{tia}[H(B_t)]}{F_i-(r-p.F_i).H(B_t)}$$

The last fraction of the second member of this expression is, from equation (4.1), equal to 1. Therefore:

$$\underset{\text{Total}}{\text{tir}\ [F_i-(r-p.F_i).H(B_t)]} = -(r-p.F_i) \qquad (4.2)$$

Thus, in the time interval T_i or (t_i, t_{i+1}), as the function $F_i-(r-p.F_i).H(B_t)$ will follows an EXPONENTIAL MODEL the quantity $-(r-pF_i)$ is supposed to be constant.

5 - Some properties of the exponential models

The exponential models are frequently used in scientific models due to its convenient properties. Some of the more useful ones are, in sections 5.1 and 5.2, applied to the model defined by equation (4.2).

5.1 - General expression

$$F_i - (r - pF_i).H(B_t) = [F_i - (r - p.F_i).H(B_i)].e^{-(r-pF_i).(t-t_i)} \qquad (5.1)$$

5.2 - Value of the function at the end of the interval T_i

$$F_i - (r - pF_i).[H(B_{i+1})] = [F_i - (r - pF_i).H(B_i)].e^{-(r-pF_i)T_i} \qquad (5.2)$$

Then the final value, $H(B_{i+1})$, will be:

$$H(B_{i+1}) = e^{-(r-pF_i)T_i}.H(B_i) + (1 - e^{-(r-pF_i)T_i}).\frac{F_i.T_i}{(r - pF_i)T_i} \qquad (5.3)$$

6 - Definition of "mean biomass ",($\bar{B}_i$), through the function $H(B_t)$, in the interval T_i

"Mean biomass", $\bar{B}_i$, in the interval T_i through the function $H(B_t)$ is the value of biomass, which verifies:

$$H(\bar{B}_i) = \frac{H(B_i) + H(B_{i+1})}{2} \qquad (6.0)$$

Particular cases:

a) When $H(B_i)=\ln(B_i)$, then $\bar{B}_i$ is the arithmetic mean of $\ln(B_i)$ and $\ln(B_{i+1})$, that is, the geometric mean between B_i and B_{i+1}.

b) When $H(B_i)= (B_i)^{-1}$, then $\bar{B}_i$ is the arithmetic mean of $(B_i)^{-1}$ and $(B_{i+1})^{-1}$, that is, the harmonic mean between B_i and B_{i+1}.

c) When $H(B_i)= (B_i)^{-p}$, then $\bar{B}_i$ is the arithmetic mean of $(B_i)^{-p}$ and $(B_{i+1})^{-p}$, that is, the mean of order (-p) between B_i and B_{i+1}.

6.1 - Relationship between $H(\overline{B_{i+1}})$ and $H(\bar{B}_i)$

For simplicity consider $T_i=T_{i+1}=T$.

In the annex 1, equation (D), the expression below is obtained:

$$H(\overline{B_{i+1}}) = \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.e^{-(r-p.F_i)T}.H(\bar{B}_i) + \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.\frac{1-e^{-(r-pF_i)T}}{(r-pF_i)T}.\frac{F_i.T}{2} + \frac{1-e^{-[r-p.(F_{i+1})]T}}{[r-p.(F_{i+1})]T}.\frac{(F_{i+1}).T}{2} \qquad (6.1)$$

6.2 - Relationship between $H(B_i)$, value at the beginning of the interval, and $H(\bar{B}_i)$, value during the interval.

In the annex 1, equation (B), the expression below is obtained:

$$H(B_i) = 2.\frac{H(\bar{B}_i)}{1+e^{-(r-p.F_i)T}} - \frac{1}{1+e^{-(r-p.F_i)T}}.\frac{1-e^{-(r-p.F_i)T}}{(r-p.F_i).T}.F_i.T \qquad (6.2)$$

6.3 - Approximate expression between $H(\overline{B_{i+1}})$ and $H(\bar{B}_i)$

In annex 2 the approximate expression between $H(\overline{B_{i+1}})$ and $H(\bar{B}_i)$ is obtained from equation (6.1), using the approximation:

$$\frac{1+e^{-(r-pF_{i+1}).T_i}}{1+e^{-(r-pF_i).T_i}} = 1$$

The equation is the following one:

$$H(\overline{B_{i+1}}) = e^{-(r-pF_i)}.H(\bar{B}_i) + \frac{1-e^{-rT}}{rT}.\left(\frac{e^{p.F_i.T}-1}{2p} + \frac{e^{p.F_{i+1}.T}-1}{2p}\right) \qquad (6.3)$$

7 - GENPROD MODEL

To obtain the GENPROD model, expression (6.1), in terms of $\overline{B}_i$, it will be used the transformation function:

$$H(Bi) = \frac{\left(\frac{B_i}{K}\right)^{-p} - 1}{p}, \quad \text{with } p>0 \qquad (7.0)$$

7.1 - Relation between $(\overline{B_{i+1}})$ and $(\overline{B}_i)$

Substituting equation (7.0) in equation (6.1) will become:

$$(\overline{B_{i+1}})^{-p} = \left[1 - \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.e^{-(r-p.F_i)T}\right].K^{-p} + \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.e^{-(r-p.F_i)T}.(\overline{B}_i)^{-p} +$$

$$+\frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.\frac{1-e^{-(r-p.F_i)T}}{(r-p.F_i)T}.K^{-p}.\frac{p.F_i.T}{2} + \frac{1-e^{-[r-p.(F_{i+1})]T}}{[r-p.(F_{i+1})]T}.K^{-p}.\frac{p.(F_{i+1}).T}{2} \qquad (7.1)$$

7.2 - Relation between B_i and $(\overline{B}_i)$

Substituting the GENPROD equation, eq. (7.0), in equation (6.2) will become:

$$B_i^{-p} = K^{-p} + \frac{1}{\left(1+e^{-(r-pF_i)T}\right)}.\left[2.(\overline{B}_i^{-p} - k^{-p}) - \frac{(1-e^{-(r-pF_i)T})}{(r-pF_i)T}.p.K^{-p}.\frac{F_iT}{2}\right] \qquad (7.2)$$

7.3 - Relation between indices of abundance, U_{i+1} and U_i

Let suppose that $U_i = q.\overline{B}_i$ and $q.f_i = F_i.T$, where q is the capturability coefficient and f_i is the fishing effort.

In terms of U the eq. (7.1) will become:

$$(\overline{U_{i+1}})^{-p} = \left[1 - \frac{1+e^{-rT+p.q.(f_{i+1})}}{1+e^{-rT+p.q.f_i}}.e^{-rT+p.q.f_i}\right].(qK)^{-p} + \frac{1+e^{-rT+p.q.(f_{i+1})}}{1+e^{-rT+p.q.f_i}}.e^{-rT+p.q.f_i}.(\overline{U}_i)^{-p} +$$

$$+\frac{1+e^{-rT+p.q.(f_{i+1})}}{1+e^{-rT+p.q.f_i}}.\frac{1-e^{-rT+p.q.f_i}}{rT-p.q.f_i}.(qK)^{-p}.\frac{p.q.f_i}{2} + \frac{1-e^{-rT+p.q.(f_{i+1})}}{rT-p.q.(f_{i+1})}.(qK)^{-p}.\frac{p.q.f_{i+1}}{2} \qquad (7.3)$$

7.4 - Approximate equation

Making:

p>0 and

$$H(B_t) = \frac{\left(\frac{K}{B_t}\right)^{p} - 1}{p}$$

from the approximate equation (6.3) the equation (7.1) will become:

$$(\overline{B_{i+1}})^{-p} = (1-e^{-(r-pF_i)T}).K^{-p} + e^{-(r-pF_i)T}.(\overline{B}_i)^{-p} + K^{-p}.\frac{1-e^{-rT}}{rT}.$$

$$.\left[\frac{e^{pF_iT}-1}{2} + \frac{e^{p(F_i+1)T}-1}{2}\right] \qquad (7.4)$$

7.5 - Approximate equation in terms of U

To obtain the equation in terms of indices (U), it is enough to replace $\overline{B}_i$ by U_i/q and FiT by qfi in expression (7.4):

$$(U_{i+1})^{-p} = (1-e^{-rT+p.q.f_i}).(qK)^{-p} + e^{-rT+p.q.f_i}.(U_i)^{-p} + (qk)^{-p}.\frac{1-e^{-rT}}{rT}.$$

$$.\left[\frac{e^{p.q.f_i}-1}{2} + \frac{e^{pq(f_i+1)}-1}{2}\right] \qquad (7.5)$$

8 - SCHAEFER MODEL

To obtain the Schaefer expressions it is enough to put p=1 in the GENPROD equations (section 7).

8.1 - Relation between $(\overline{B_{i+1}})$ and $(\overline{B}_i)$

From equation (7.1) the Schaefer equation will be:

$$(\overline{B_{i+1}})^{-1} = \left[1 - \frac{1+e^{-[r-(F_{i+1})]T}}{1+e^{-(r-F_i)T}}.e^{-(r-F_i)T}\right].K^{-1} + \frac{1+e^{-[r-(F_{i+1})]T}}{1+e^{-(r-F_i)T}}.e^{-(r-F_i)T}.(\overline{B}_i)^{-1} +$$

$$+\frac{1+e^{-[r-(F_{i+1})]T}}{1+e^{-(r-F_i)T}}.\frac{1-e^{-(r-F_i)T}}{(r-F_i)T}.K^{-1}.\frac{F_i.T}{2} + \frac{1-e^{-[r-(F_{i+1})]T}}{[r-(F_{i+1})]T}.K^{-1}\frac{(F_{i+1}).T}{2} \qquad (8.1)$$

8.2 - Relation between B_i and $(\overline{B}_i)$

From equation (7.2):

$$B_i^{-1} = K^{-1} + \frac{1}{\left(1+e^{-(r-F_i)T}\right)}.\left[2.(\overline{B}_i^{-1} - K^{-1}) - \frac{(1-e^{-(r-F_i)T})}{(r-F_i)T}.K^{-1}.\frac{F_iT}{2}\right] \qquad (8.2)$$

8.3 - Relation between indices of abundance, U_{i+1} and U_i

From equation (7.3):

$$(U_{i+1})^{-1} = \left[1 - \frac{1+e^{-rT+q.(f_{i+1})}}{1+e^{-rT+q.f_i}}.e^{-rT+q.f_i}\right].(qK)^{-1} + \frac{1+e^{-rT+q.(f_{i+1})}}{1+e^{-rT+q.f_i}}.e^{-rT+q.f_i}.(U_i)^{-1} +$$

$$+\frac{1+e^{-rT+q.(f_{i+1})}}{1+e^{-rT+q.f_i}}.\frac{1-e^{-rT+q.f_i}}{rT-q.f_i}.(qK)^{-1}.\frac{q.f_i}{2} + \frac{1-e^{-rT+q.(f_{i+1})}}{rT-q.(f_{i+1})}.(qK)^{-1}.\frac{q.(f_{i+1})}{2} \qquad (8.3)$$

8.4 - Approximate equation

From equation (7.4):

$$(\overline{B_{i+1}})^{-1} = (1-e^{-(r-F_i)T}).K^{-1} + e^{-(r-F_i)T}.(\overline{B}_i)^{-1} + K^{-1}.\frac{1-e^{-rT}}{rT}.$$

$$\left[\frac{e^{F_iT}-1}{2} + \frac{e^{(F_{i+1})T}-1}{2}\right] \qquad (8.4)$$

8.5 - Approximate equation in terms of U

From equation (7.5):

$$(U_{i+1})^{-1} = (1-e^{-rT+q.f_i}).(qK)^{-1} + e^{-rT+q.f_i}.(U_i)^{-1} + (qk)^{-1}.\frac{1-e^{-rT}}{rT}.$$

$$.\left[\frac{e^{qf_i}-1}{2} + \frac{e^{q(f_{i+1})}-1}{2}\right] \qquad (8.5)$$

9 - FOX MODEL

9.1 - Relation between $(\overline{B_{i+1}})$ and $(\overline{B}_i)$

To obtain this equation it is preferable to put p=0 and $H(\overline{B}i) = \ln(k)-\ln(\overline{B}i)$ in expression (6.1).Thus it will be:

$$\ln(\overline{B_{i+1}}) = (1-e^{-rT}).\ln(K) + e^{-rT}\ln(\overline{B}_i) - \frac{1-e^{-rT}}{rT}.(F_i + F_{i+1}).\frac{T}{2} \qquad (9.1)$$

9.2 - Relation between B_i and $(\overline{B}_i)$

To obtain this equation it is preferable to put p=0, $H(B_i)=\ln K-\ln(B_i)$ and $H(\overline{B}_i)=\ln(k)-\ln(\overline{B}_i)$ in expression (6.2).Thus it will be:

$$\mathrm{Ln}(B_i)=\frac{1}{1+e^{-rT}}\cdot\left[(e^{-rT}-1).\mathrm{Ln}(K)+2\mathrm{Ln}(\overline{B}_i)+(1-e^{-rT}).\frac{F_iT}{rT}\right] \qquad (9.2)$$

9.3 - Relation between indices of abundance, U_{i+1} and U_i

Substituting in equation (9.1) $\overline{B}i$ by Ui/q and FiT by qfi it will become the Fox equation in terms of U:

$$\ln(U_{i+1})=(1-e^{-rT}).\ln(qK)+e^{-rT}.\ln(U_i)-\frac{1-e^{-rT}}{rT}.\frac{q.[f_i+(f_{i+1})]}{2} \qquad (9.3)$$

9.4 - Approximate equation

The Fox approximated equation will be the same than equation (9.1). Thus it will be:

$$\ln(\overline{B_{i+1}})=(1-e^{-rT}).\ln(K)+e^{-rT}\ln(\overline{B}_i)-\frac{1-e^{-rT}}{rT}.(F_i+F_{i+1}).\frac{T}{2} \qquad (9.4)$$

9.5 - Approximate equation in terms of U

As for the approximated Fox equation in terms of $\overline{B}i$, the approximated Fox equation in terms of U will be the same that the non approximated equation, eq. (9.3). Thus:

$$\ln(U_{i+1})=(1-e^{-rT}).\ln(qK)+e^{-rT}.\ln(U_i)-\frac{1-e^{-rT}}{rT}.\frac{q.[f_i+(f_{i+1})]}{2} \qquad (9.5)$$

ANNEX 1 - Derivation of the relationship between $H(\overline{B_{i+1}})$ and $H(\overline{B}_i)$.

From equation (6.0) and (5.3) it will be:

$$H(\overline{B}_i)=(1+e^{-(r-pFi)T}).\frac{H(B_i)}{2}+\frac{1-e^{-(r-pF_i)T}}{(r-pF_i)T}.\frac{F_iT}{2} \qquad (A)$$

but solving that equation (A) in order to $H(B_i)$ it will be:

$$H(B_i)=2.\frac{H(\overline{B}_i)}{1+e^{-(r-pF_i)T}}-\frac{1}{1+e^{-(r-pF_i)T}}.\frac{1-e^{-(r-pF_i)T}}{(r-pF_i)T}.F_iT \qquad (B)$$

This equation permits to obtain $H(B_i)$, value at the beginning of the interval Ti, in function of $H(\overline{B}_i)$, value during the interval, and therefore to obtain the initial biomass (B_i) from the mean biomass, $(\overline{B}_i)$.

Calculating the expression of $H(\overline{B}_i)$ before, for the next interval, T_{i+1}, it will be:

$$H(\overline{B_{i+1}})=(1+e^{-(r-pF_{i+1})T}).\frac{H(B_{i+1})}{2}+\frac{1-e^{-(r-pF_{i+1})T}}{[r-p(F_{i+1})T]}.\frac{(F_{i+1})T}{2}$$

Substituting $H(B_{i+1})$, in this equation, by the value given in eq.(5.3) it will be:

$$H(\overline{B_{i+1}})=\frac{1+e^{-[r-p(F_{i+1})T]}}{2}.e^{-(r-pF_i)T}.H(B_i)+\frac{1+e^{-[r-p(F_{i+1})T]}}{2}.\frac{1-e^{-(r-pF_i)}}{(r-pF_i)}.F_iT+$$

$$+\frac{1-e^{-[r(-p(F_{i+1})T]}}{(r-p.F_{i+1})T}.\frac{(F_{i+1})T}{2} \qquad (C)$$

Replacing now the value of $H(B_i)$, in this equation, by the expression obtained before in terms of $H(\overline{B}_i)$, (eq. B) it will be:

$$H(\overline{B_{i+1}}) = \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.e^{-(r-p.F_i)T}.H(\overline{B}_i) + \frac{1+e^{-[r-p.(F_{i+1})]T}}{1+e^{-(r-p.F_i)T}}.\frac{1-e^{-(r-pF_i)T}}{(r-pF_i)T}.F_i.\frac{T}{2} +$$

$$+\frac{1-e^{-[r-p.(F_{i+1})]T}}{[r-p.(F_{i+1})]T}.(F_{i+1}).\frac{T}{2} \qquad \text{(D)}$$

ANNEX 2 - Approximate relationship between $H(\overline{B_{i+1}})$ and $H(\overline{B}_i)$.

If $\frac{1+e^{-[r-p.(Fi+1)]T}}{1+e^{-(r-p.Fi)T}}$ be approximated by 1, then equation (D) of the annex 1, can be written in the following approximate form:

$$H(\overline{Bi+1}) = e^{-(r-p.Fi)T}.H(\overline{B}i) + \frac{1-e^{-(r-p.Fi)T}}{(r-p.Fi)T}.\frac{FiT}{2} + \frac{1-e^{-[r-p.(Fi+1)]T}}{[r-p.(Fi+1)]T}.\frac{(Fi+1)T}{2}$$

Using the known approximation $\frac{1-e^{-X}}{X} = e^{-\frac{X}{2}}$, for small values of X, the equation before becomes:

$$H(\overline{Bi+1}) = e^{-(r-p.Fi)T}.H(\overline{B}i) + e^{-(r-p.Fi)\frac{T}{2}}.\frac{FiT}{2} + e^{-[r-p.(Fi+1)]\frac{T}{2}}.\frac{(Fi+1)T}{2}$$

or, putting the common factor $e^{-\frac{rT}{2}}$ in evidence in the last two terms, it will be:

$$H(\overline{Bi+1}) = e^{-rT+p.FiT}.H(\overline{B}i) + e^{-\frac{rT}{2}}.\left[e^{\frac{pFiT}{2}}.\frac{FiT}{2} + e^{\frac{p(Fi+1)T}{2}}.\frac{(Fi+1)T}{2}\right]$$

Using again the approximation anterior, but backwards, that is $e^{-\frac{X}{2}} = \frac{1-e^{-X}}{X}$ and $e^{\frac{X}{2}} = \frac{e^{X}-1}{X}$ it will be:

$$H(\overline{B_{i+1}}) = e^{-rT+pF_iT}.H(\overline{B}_i) + \frac{1-e^{-rT}}{rT}.\left[\frac{e^{pF_iT}-1}{2p} + \frac{e^{p(F_{i+1})T}-1}{2p}\right]$$

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