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AN IMPLEMENTATION OF FOX'S PRODUCTION MODEL WITH MIXING: INITIAL RESULTS

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SUMMARY

Production models incorporating mixing among stocks have been discussed previously. However, there have been few published attempts to fit such models to data. This report describes implementation of such a model and initial results in fitting simulated data. It appears that such an approach is unlikely to be useful for estimating mixing rates; however, it might prove useful for sensitivity analyses of the possible effects of mixing at various rates.

RESUME

Les modèles de production qui présentent des mélanges entre les stocks ont été discutés antérieurement. Cependant, il y a eu quelques tentatives publiées pour ajuster ces modèles aux données. Ce rapport décrit l'application d'un tel modèle et les premiers résultats pour ajuster les données simulées. Il semble peu probable qu'une telle tentative soit utile pour estimer les taux de mélange; néanmoins, elle pourrait s'avérer utile dans les analyses de sensibilité d'effets éventuels de mélange à différents taux.

RESUMEN

Se discutieron previamente los modelos de producción que incorporaban mezcla entre stocks. Sin embargo, se han publicado pocos intentos para ajustar tales modelos a los datos. Este informe describe la implementación de un modelo de esas características y los resultados iniciales del ajuste de datos simulados. Parece improbable que un enfoque de ese tipo pueda ser útil para estimar las tasas de mezcla; sin embargo, podría demostrar su utilidad en análisis de sensibilidad de los posibles efectos de mezclas en varias proporciones.

INTRODUCTION

To date, about 20 tagged yellowfin tuna are known to have traversed the Atlantic Ocean from west to east. The proportion of fish that moves like this is unknown, as is the proportion of fish, if any, that moves in the opposite direction. A general but useful hypothesis about the underlying stock structure is that yellowfin tuna in the Atlantic form two or more groups (stocks) that have a limited but biologically significant amount of mixing. Fox (1975, 1977) formulated a model that might describe such a stock structure, and he demonstrated mathematically some consequences to population dynamics that would result from such mixing. However, Fox did not attempt to fit his model to data. This paper describes an implementation of Fox's model and some very preliminary results in fitting simulated data.

The term "stock" is used in this paper in a way slightly different from the common usage. Here, it means a group of fish that has some degree of mixing with another group or groups. The mixing rate might be zero or it might be larger than zero.

IMPLEMENTATION

Pella (1967) presented the equations for a continuous-time production model based on the logistic population model. Prager (1992, 1994) extended Pella's work and implemented the model in a FORTRAN program (ASPIC). For this study, the Fox mixing model was implemented as a modification of ASPIC with the following characteristics:

1. Two stocks were assumed (although the model could be extended to more than two stocks). Each stock is described by five parameters specific to that stock and its fishery: initial biomass B_1, intrinsic rate of increase r, carrying capacity K, catchability coefficient q, and transfer (mixing) coefficient T.
2. As in the original production model, natural mortality, growth, and recruitment are lumped together into the parameter r. Except for mixing, population dynamics (including fishing mortality) are considered to occur in continuous time.
3. No assumption of equilibrium conditions is used.
4. For computational simplicity, mixing is modeled as an instantaneous process occurring at the end of each year. Mixing is assumed to follow the Fox mixing model (described next).

The Fox mixing model states that the annual transfer of biomass M_{ij} from stock i to stock j depends on the stock sizes and the carrying capacity of the second stock. Fox proposed the following equation:

$$(1) \quad M_{ij} = T_{ij} B_i \left(\frac{K_j - B_j}{K_j} \right),$$

where M_{ij} is the transfer (mixing) in biomass units, B_i and B_j are the current biomass levels of the two stocks, K_i and K_j are the carrying capacities of the two stocks, and T_{ij} is an unknown constant that describes the magnitude of movement from stock i to stock j. The quantities M and B vary with time, but subscripts denoting time are omitted here.

Fox suggested his model as a simple representation of a plausible hypothesis, and did not claim it had a biological basis.

A modified version of Fox's equation would have the strength of mixing more strongly dependent on the relative size of the stock of origin:

$$(2) \quad M'_{ij} = T_{ij} B_i \left(\frac{B_i}{K_i} \right) \left(\frac{K_j - B_j}{K_j} \right)$$

This alternative equation is not examined further here. Mullen (1989) developed a production model with mixing based on a diffusion model, which provides a third alternative.

APPLICATION, RESULTS, DISCUSSION

Several simulated data sets were generated conforming exactly to the proposed model, but with multiplicative normally distributed random error added to the "true" effort to obtain the "observed" effort. The modified ASPIC computer program described above was then used to estimate the 10 unknown parameters. The purpose of this exercise was to ascertain whether this approach could be used to estimate population parameters without prior knowledge of the mixing rates. If so, then the model might be useful for estimating mixing rates and MSY for populations with unknown mixing rates.

A simulating fishing history (Table 1) was chosen to provide contrast between stocks, and simulated data were generated from this history. With 5% or 10% noise (i.e., a random normal multiplier had s.d. equal to 5% or 10% of the true effort) in the effort data, the computer program could not find a solution, and no estimates could be obtained. (The unmodified single-stock ASPIC model can easily estimate parameters on data with such levels of error.) Estimation was then attempted on a data set generated with only 1% noise in effort, and estimates were obtained, although the computations took several hours of computer time.

Results from this exercise (Table 2), suggest that it may be impractical to estimate mixing rates by this method. The mixing rates T were estimated less well than other model parameters. The two catchability coefficients q were estimated fairly well; the small (1%) level of random error probably helped in this regard. The same is true of the starting biomasses B_1 (which for technical reasons were estimated as ratios of B_1 to B_{MSY}).

Estimates of the individual parameters r and K from production models are typically highly correlated; even when r and K are not estimated precisely, the estimate of MSY, which depends on both of them, can be quite precise. In this test, the estimates of MSY were much worse than would be expected from a model of a single simulated stock with so little error. However, the total estimated MSY from both stocks was 10 213 MT, quite close to the sum of the true values (10 000 MT). Whether this is due to chance remains to be seen. However, if a model of this type can estimate a basin-wide MSY with reasonable precision, it may be useful, even if it cannot estimate mixing rates.

In a final simple test, it was assumed that mixing rates were known *a priori*. With knowledge of the mixing rates, it was possible to recover the other parameter values without difficulty (Table 3). Thus if the true stock dynamics are sufficiently close to the proposed model, and mixing rates can be estimated by some other means (perhaps tagging), it seems possible to estimate management benchmarks using the techniques described here.

Production models are attractive in general because of their very simple parameterization. That is true of variations such as this. However, the simple mixing model presented here is not intended to replace more complex models such as that of Fonteneau (1992), and indeed neither approach should be thought of as "better" or "worse" than the other. More complex models offer the potential to achieve greater biological realism, generally at the cost of higher demand for data (e.g., estimates of age-specific migration rates and other age-specific quantities). Simple models can often provide parameter estimates from fewer data, at the cost of stronger assumptions. In the author's opinion, simple and complex methods are complementary tools in fish stock assessment.

This short paper is presented as a progress report only, and the results are quite preliminary. More experience with this type of model should produce more definitive conclusions.

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Table 1. Simulated fishing history by stock, used to generate 16 years of simulated data. The true value of F_{MSY} is 1.0 for each stock.

Year	F, Stock 1	F, Stock 2
1961	0.6	0.9
1962	0.75	1.2
1963	0.85	0.96
1964	0.8	1.3
1965	1.05	1.2
1966	0.85	1.05
1967	0.75	1.1
1968	0.85	1.2
1969	0.75	0.8
1970	1.5	0.9
1971	1.8	0.75
1972	1.4	0.68
1973	1.7	0.69
1974	2	0.55
1975	1.55	0.48
1976	1.43	0.45

Table 2. True and estimated parameters for two populations, estimated from simulated data described in text and Table 1, with 1% normally distributed error added. In this case, all parameters were estimated.

Parameter	Stock 1		Stock 2	
	True	Estimated	True	Estimated
B_1:B_{MSY}	1.20	1.08	0.60	0.784
K	10 000	7 876	10 000	9 800
r	2.0	3.31	2.0	1.48
q	1.0×10^{-4}	1.21×10^{-4}	1.0×10^{-4}	1.07×10^{-4}
T	0.15	0.427	0.25	1.15×10^{-9}
MSY	5 000	6 514	5 000	3 699

Table 3. True and estimated parameters for two populations, estimated from simulated data described in text and Table 1, with 1% normally distributed error added. In this case, the mixing parameters T were assumed known and the other parameters were estimated.

Parameter	Stock 1		Stock 2	
	True	Estimated	True	Estimated
B_1:B_{MSY}	1.20	1.21	0.60	0.682
K	10 000	9 671	10 000	9 787
r	2.0	2.07	2.0	2.05
q	1.0×10^{-4}	1.02×10^{-4}	1.0×10^{-4}	1.01×10^{-4}
T	0.15	——	0.25	——
MSY	5 000	5 008	5 000	4 995