

## A COMPARISON OF THREE METHODS TO CONVERT CATCH AT LENGTH DATA INTO CATCH AT AGE

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### SUMMARY

Three methods for estimating catch at age data from length frequencies are compared. The first method is the familiar cohort slicing which presumes a fixed size range for each age and partitions the catch at length accordingly. The second method is Kimura-Chikuni, which iteratively adjusts numbers in each age class until their combined length distribution matches that of the catch at length. The final method iteratively estimates age-length keys from a growth model by estimating cohort strength to update the key then using the key to update the catch at age. These methods are tested using data generated from beta distributions. The performance of the methods are tested for measurement noise and model mis-specification.

### RESUME

Trois méthodes d'estimation des données de prise par âge à partir des fréquences de taille sont comparées. La première méthode est la méthode bien connue du découpage qui postule une gamme fixe de tailles pour chaque âge, et répartit en conséquence la prise par taille. La deuxième méthode est celle de Kimura-Chikuni, qui ajuste itérativement le nombre de chaque classe d'âge jusqu'à ce que leur distribution combinée de taille concorde avec celle de la prise par taille. La dernière méthode estime itérativement des clés âge-taille à partir d'un modèle de croissance en estimant la magnitude de la cohorte pour actualiser la clé, puis en utilisant cette dernière pour actualiser la prise par âge. Ces méthodes sont testées en utilisant les données découlant des distributions beta. La performance des méthodes est testée du point de vue bruitage et misspécification des modèles.

### RESUMEN

Se comparan tres métodos para estimar los datos de captura por edad partiendo de frecuencias de tallas. El primer método es el ya familiar de corte de cohortes, que presupone una gama fija de tallas para cada edad y que reparte la captura por talla en consecuencia. El segundo método es el de Kimura-Chikuni, que ajusta de forma iterativa los números en cada clase de edad hasta que su distribución de tallas combinadas concuerda con la de la captura por talla. El último de los métodos, estima de forma iterativa las claves edad-talla en base a un modelo de crecimiento, por medio de la estimación de la fuerza de la cohorte con el fin de actualizar la clave, y después, usando la clave para actualizar la captura por edad. Estos métodos se prueban usando datos generados partiendo de distribuciones beta. La actuación de los métodos se prueba en relación con las perturbaciones en la medición y especificación errónea del modelo.

## Introduction

There are many stocks for which annual age length keys (ALKs) are impossible, or too prohibitively expensive, to obtain. ALKs combine growth information with the relative numbers at age. For this reason it is in general impossible to apply an ALK to other years or fleet segments which do not have aged subsamples. (Westrheim & Ricker, 1978). Jones (1974) assumed that the numbers at age were stable (at least constant over the period of investigation) in order to use a growth model to infer numbers at age from catch length distributions. Another approach is to use a statistical method to separate the catch at length into age classes (MacDonald & Pitcher, 1979) and then proceed with traditional age based sequential population analysis (SPA). In practice this approach requires heavily constrained models because of the low information content in the length frequencies found in most fisheries data.

In this paper we shall investigate three methods which concentrate on the conversion of catch at length (CAL) to catch at age (CAA). Two of these methods, Cohort Slicing and Kimura - Chikuni, are well known and tried. They operate directly on a catch at length matrix and convert it into catch at age. The Cohort Slicing method uses only mean size at age information while Kimura-Chikuni method includes distributional information at size. The third method is new and is a simplification of methods the author has presented before (Mohn & Savard, 1989 and Mohn, 1991). The principal simplification is that the SPA is strictly age based. We will call this method SP- Key to denote that it iteratively finds virtual populations and age length keys.

The first part of the work describes a growth model based on beta functions. It is important to have an underlying growth model which is more complicated than any of the analysis routines otherwise it may be too easy to test the power of the analytical methods. The growth model considers the numbers at a given size, age and year; it determines the survivors and then distributes them with a beta function over a range of sizes for the next age and year.

Four simulation experiments are performed. The data are meant to be difficult; there is considerable overlap in the older ages and strong year classes are mixed among weaker ones. In the first three experiments coarse length information is used - only 12 size classes. This is done to allow stochastic replicates to be performed on a microcomputer, generally 20 replicates. The fourth experiment doubles the number of length classes to see if such coarse data were prejudicial to one of the methods.

## Methods

### 1) Data generation

#### 1.a) Growth model

A stock projection model was developed which generates numbers at size and age. Growth from a given size and age is described by a beta function. The beta function has a finite range and is sufficiently versatile to describe a wide range of behaviour which is controlled by two parameters. Figure 1 shows sample beta functions for 3 pairs of parameter values. We have constrained the parameters to be integers and when  $p$  and  $q$  are equal the distribution is symmetric. For the simulations below parameter values of  $p = q = 3$  are used. The surviving animals in a particular length-age-year cell ( $N_{l,a,y}$ ) are distributed over lengths for the following year and age as

$$N_{\cdot,a+1,y+1} = N_{l,a,y} \times \text{Beta}(p,q) \quad (1)$$

where the dot subscript denotes all values of the subscript.

The survivorship is the usual exponential model wherein natural and fishing mortalities are matrices of length by age. In the versions of the model used in this study the natural mortality is 0.2 for all ages and sizes and the selectivity is an explicit function of length alone which is multiplied by a fully recruited  $F$  for each year. See Table 1.

The model uses a mean size and distribution range at age (Table 1). To start the projections, the recruits are distributed using the beta distribution for age 1. Similarly in for the first year of the projection the numbers at age are distributed using the respective beta parameters at age. The size structure then takes a few years to smear out as the initial distributions propagate in time.

The projection routine is written in C and an interface is an expanded XTalk script which is used to control the data generation and analysis. Unfortunately the XTalk interface does not yet support 3 dimensional matrices so the numbers at length x age x year and the catch at length x age x year are projected onto length by year matrices for input for conversion into catch at length by the three methods under review. Also, they are projected onto age by year matrices for comparison with the outputs from the three methods. A final summary matrix was the mean size at length over the duration of the projection. This matrix was normalized such that the total over all length groups for a given age is 1 and we will denote it as the growth template. This size at age information in the growth template is used by each of the catch conversion routines in their own manner.

The resultant catch at age was multiplied by a weight at age vector to give biomass. The biomass from ages 2 to 4 was used as a signal for SPA tuning.

#### 1.b) Error structure (observational + model)

The observational error was simulated by adding lognormal noise to the catch at length. Noise levels typically ranged from 0 to 0.6. Noise was not added to the abundance index to which the SPA was tuned. The only model parameter that was varied was the selectivity at age used in ADAPT. For experiments 2-4 it was estimated from a linear interpolation of the true selectivity at length onto the mean size at age. The interpolated values are 0.05, 0.75, 1 1 1. In Experiment 1 arbitrary values were used and are given in Table 1.

### 2) Analytical programs

#### 2.a) Cohort slicing

Cohort slicing takes the length distributions for each year and slices it into ages. The method used for this study was to take the mean size at age from the growth template. Halfway between these means were chosen as slicing points. Thus a slice is made at 45 cm and all fish less than this are

called 1 year olds and all fish from that slice to the next one at 65 cm are called 2s, etc. This method ignores the dispersions at each age. Its main advantages are that it is very simple and fast to execute. The catch at age for this method using parameters from Experiment 1, with no observational noise, are given in Table 2a.

#### 2.b) Kimura-Chikuni

Kimura & Chikuni (1987) outline a method for iteratively determining the age structure from a length sample using the key from a different sample. Their method corrects for the problem (See Westrheim & Ricker. 1978) of having a aged sample from one year or fleet and applying it to other years, etc. In effect, the method iteratively adjusts the strengths of the numbers in the aged sample until the lengths from that sample match the length distribution to be aged. The method is computationally compact but convergence is slow. In this study the iteration limit was set at 200 and was often the cause of termination.

The aged sample is the growth template mentioned above. It is an average over the 9 years of the simulation and in contrast to Cohort Slicing it uses the distributional information for each age. The catch at age for this method using parameters from Experiment 1, with no observational noise, are given in Table 2b.

#### 2.c) SP - Key

This method is a simplification of techniques presented in Mohn & Savard (1989) and Mohn (1991). The former of these is a full three dimensional (length x age x year) analysis and the second tunes to length based data. SP-Key separates the conversion of CAL to CAA and the SPA functions inside a single iteration. The core of the approach is to estimate an age length key for each year from the growth template. Figure 2 is a schematic displaying the general approach. The input data are catch at length and an abundance index, the age aggregated 2+ biomass. To start the process an initial conversion to catch at age is done by Cohort Slicing or Kimura

Chikuni. As the results were not affected by the starting catch at age, Cohort Slicing was chosen because of its speed. These catches are converted into numbers at age by a version of ADAPT described below, but any SPA could be used. The numbers at age are then used to weight the growth template to form an ALK for each year. Resultant ALKs convert the catches at length into new estimates of catch at age and the model iterates until convergence. The iterations stop when the maximum change in a catch at age is less than 1%.

SP-Key executes much more slowly than Cohort Slicing or Kimura-Chikuni as each iteration runs an ADAPT to convergence. Although few iterations are required, the limit is 6 for this study, each is a full ADAPT. The catch at age for this method using parameters from Experiment 1, with no observational noise, are given in Table 2c.

#### 2.d) C version of ADAPT

A special version of the ADAPT framework (Gavaris, 1991) was used to tune the catch at age from the respective methods. Because the bulk of the computations are executed in a compiled version of C, attempts were made to incorporate some of the adaptability of the APL based ADAPT. The tuning information is assumed to be a number of time series (vectors). They could be catch rates in numbers of biomass, or efforts. Each vector covers the time span of the catch matrix. Minus values signal that a particular year, or years, is not to be used. An accompanying vector of four elements contains information as to the which model data the tuning is to be done. The first element signals whether numbers, biomass or F's are required. The next two determine the age range and the final element is the month, presuming that the catch data are annual. A selectivity pattern is specified and a single terminal F is applied over a specified age range. F at the oldest age was set initially at 0.6 for all years and updated within ADAPT. If a new terminal F did not match the previous on the SPA was run with an average F over younger ages until the pattern stabilized.

For this analysis the 2+ biomass was used for tuning. ADAPT was a two parameter model, the F in the last year and catchability,  $q$ , for the biomass

vector. Such a sparse model was chosen to minimize computational requirements. Also, this study is a comparison of methods to determine catch at age and not to find an optimal APAPT configuration

#### Results

Table 3 contains the catch at length from Experiment #1 which is then passed to each of the methods for conversion into catch at age. The later years show only a single smeared mode. Table 3 also contains the numbers and catch at age from the projection which will be used as references in assessing performance.

Table 4 contains sample conversion into catch at age for each of the methods. No noise has yet been added to the CAL. The estimate using SP-Key requires and estimate of NAA and FAA. In Table 4c the true NAA is used and the FAA is approximated by a value of 0.6 for all ages and years. It is interesting to note how each of the methods deals with the strong year 1981 year class. Cohort slicing underestimates and greatly overestimates the 1982 year class in 1983. Both K-C and SP-Key overestimate the 1981 yearclass in 1983.

Table 5 is the residuals from Table 4. Table 4a shows the problem cohort slicing has with strong cohorts as the residuals display diagonal patterns. Table 4b (K-C) does not show any obvious pattern. The SP-Key results reflect that most of the errors are in ages 1 and 2, where the 1 year olds are underestimated and the 2s overestimated.

Table 6 contains the full tests in which the CAL is converted to CAA and then an SPA is run to yield NAA. This table gives the residual sum of squares from ADAPT and the mean absolute residuals for catch and numbers (MACR and MANR respectively) at age for each of the methods. Only two replicates were carried out at noise levels 0 and 0.05, and 20 replicates were used for each of the higher noise levels. These results are also summarized in Figures 3- 5.

Experiment #1 uses the parameters given in Table 1. Experiment #2 changed the mean size of 4 year olds from 85 to 90. As the mean size of 5s is 95 this should make it more difficult to resolve these two ages. Slicing actually performed a little better with these data while K-C performed a little worse in terms of MANR. SP-Key was not affected by the change.

Experiment #3 uses the same parameters and #2 except that the selectivity at age is changed to reduce the selectivity on young animals. This change had very little affect on the results. Naturally, it did not affect the MANC for slicing or K-C at all.

Experiment #4 doubled the resolution in the catch at length data. This was only tested at three noise levels. The increased resolution helped K-C and SP-Key, but had little affect on slicing. The very high residual MANR at 0.05 noise level for slicing is an artifact. Only 2 replicates are used at this noise level and one of them failed to converge. Anyone using this data would have rejected the results and tried different initial values for ADAPT.

Table 7 examines the performance of age based analysis with these data. The true catch at age from the growth model are corrupted with increasing lognormal noise and NAA is estimated using the same ADAPT model as above. Figure 5 shows these results superimposed on Experiment #3. Because the magnitude of the residuals was so similar to some of the length based results a second series was done with a second random seed.

#### Discussion.

This study is based a growth model that is more complicated than any of the methods trying to "decode" the CAL information. Hopefully, a level playing field was established for these trials. A simulation study of this sort is difficult because there are so many parameters that should be systematically investigated. For example, if within ADAPT the iterative convergence of the F matrix were turned off the cohort slicing results degraded much more than the SP-Key.

All the methods could be tuned and/or made more sophisticated. One obvious improvement would be to have annual size at age information. A second improvement would be check the ADAPT results in the stochastic replicates and remove cases where the data did not converge. Similarly the

ADAPT model used above was only a 2 parameter model. This sparse configuration was adopted to improve throughput.

In terms of the mean residual catch, cohort slicing did very well. However, the subsequent NAA were relatively poor. K-C produced fairly good numbers at age at low noise levels but performance degraded quickly as the noise increased. The SP-Key method did well and closely approximated the age-based results at higher noise levels. Subject to further testing this method seems quite promising.

Further testing should include real and simulated data. At a recent ICES meeting it was mentioned that there was a need for tested CAA data to compare SPA techniques. Analogously, some standard CAL data would also be useful in the development of length based methods. Testing against aged fisheries data is also needed.

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Tables:

Table 1. Typical parameters used in growth model and in Experiment # 1. Some of these values will change from experiment to experiment.

1.a Mean size and range at age and selectivity used in ADAPT.

Age	Size	Range	Selectivity
1	35	30	0.1
2	55	30	0.5
3	75	30	1.0
4	85	30	1.0
5	95	30	1.0

1.b Selectivity at length

Size	Selectivity
10	0.0
20	0.0
30	0.1
40	0.5
50	1.0
60	1.0
70	1.0
80	1.0
90	1.0
100	1.0
110	0.6
120	0.4

1.c Effort (fully recruited F) and recruitment series.

Year	Effort	Recruits
1	0.6	100
2	0.6	360
3	0.6	240
4	0.4	100
5	0.4	100
6	0.4	100
7	0.7	300
8	0.7	100
9	0.7	100

Table 2 Distribution of average size at age over the 9 years of the simulations.

Age	1	2	3	4	5
15	-	-	-	-	-
25	21.20	-	-	-	-
35	57.58	4.57	-	-	-
45	21.20	25.63	1.29	0.28	0.06
55	-	43.88	9.31	2.41	0.62
65	-	22.49	26.70	9.58	3.07
75	-	3.41	35.86	24.27	9.79
85	-	-	20.75	34.48	22.97
95	-	-	5.47	20.90	32.77
105	-	-	0.58	6.65	20.70
115	-	-	-	1.29	7.59
125	-	-	-	0.10	2.37

Table 3. Catch at length using the parameters in Table 1 (Experiment #1). Length classes are at the middle of each interval.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
35	3.04	11.14	8.00	2.36	2.17	2.17	10.82	4.21	3.75
45	9.01	22.60	28.65	11.54	7.02	6.87	22.60	21.49	11.41
55	19.02	14.10	49.34	27.74	13.85	11.86	17.67	45.91	20.38
65	12.26	11.70	30.00	24.14	16.84	11.61	15.73	28.44	22.05
75	17.77	10.67	11.36	16.90	17.63	11.44	13.09	11.61	18.79
85	16.52	10.13	6.79	10.33	13.92	10.97	11.29	6.92	12.11
95	8.26	6.51	4.30	4.21	7.24	7.76	7.72	4.06	5.05
105	1.75	2.31	1.82	1.19	2.55	3.94	4.08	1.79	1.54
115	0.00	0.22	0.30	0.19	0.41	0.94	1.11	0.42	0.31
125	0.00	0.00	0.02	0.02	0.05	0.19	0.25	0.09	0.06

3b. Numbers at age from Experiment #1.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
1	100.0	360.0	240.0	100.0	100.0	100.0	300.0	100.0	100.0
2	80.0	74.6	268.6	179.1	76.8	76.8	76.8	220.6	73.5
3	60.0	38.6	37.7	135.9	106.0	45.4	45.4	35.9	103.5
4	40.0	26.9	17.3	17.0	74.8	58.3	25.0	18.5	14.7
5	20.0	17.9	12.1	7.8	9.4	41.2	32.1	10.2	7.6

3c. Catch at age from Experiment #1.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	8.04	28.97	19.31	5.54	5.54	5.54	27.72	9.24	9.24
2	30.03	26.06	93.81	45.19	19.48	19.48	30.23	86.28	28.76
3	24.78	15.94	15.49	40.60	31.70	13.58	20.85	16.45	47.44
4	16.52	11.13	7.14	5.08	22.32	17.40	11.45	8.50	6.72
5	8.26	7.31	4.84	2.24	2.68	11.77	14.11	4.48	3.33

Table 4. Sample output for each method under review.

4a. Estimated catch at age from Cohort Slicing.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	11.80	33.13	35.87	13.59	9.01	8.86	32.80	25.12	14.86
2	30.23	25.19	76.95	49.65	29.11	22.44	32.35	71.93	40.41
3	24.93	15.50	16.94	23.12	24.35	16.56	18.76	17.08	25.43
4	13.75	8.98	5.99	8.24	11.69	9.99	10.18	5.99	9.71
5	6.91	6.60	4.83	4.05	7.55	9.92	10.28	4.84	5.08

4b. Estimated catch at age from Kimura - Chikuni.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	1.11	13.53	0.96	0.00	0.00	0.66	11.20	0.00	0.01
2	34.18	42.77	114.54	51.21	26.61	24.99	50.79	94.58	42.79
3	18.09	6.31	6.05	38.75	30.58	14.04	13.34	20.06	40.54
4	34.25	26.80	19.05	7.04	21.40	17.15	17.20	4.37	9.25
5	0.00	0.00	0.00	1.65	3.13	10.93	11.83	5.94	2.89

4c. Estimated catch at age from SP-Key with true numbers at age and F at age set to 0.6.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	2.36	14.63	5.41	1.25	1.75	1.76	13.18	1.95	3.06
2	30.79	37.34	105.48	47.56	21.84	22.27	42.80	92.10	33.65
3	28.11	17.68	17.08	42.34	33.03	14.55	22.56	17.67	48.76
4	18.64	12.24	7.54	5.15	22.32	17.39	11.54	8.58	6.61
5	7.72	7.52	5.08	2.34	2.78	11.79	14.29	4.65	3.41

Table 5. Residuals of estimated catch and true catch at age.

5a Cohort Slicing.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	3.7	4.1	16.5	8.0	3.4	3.3	5.0	15.8	5.6
2	0.1	-0.9	-16.9	4.4	9.6	2.9	2.1	-14.4	11.6
3	0.1	-0.5	1.4	-17.5	-7.4	2.9	-2.1	0.6	-22.1
4	-2.8	-2.2	-1.2	3.1	-10.7	-7.5	-1.3	-2.6	2.9
5	-1.4	-0.8	-0.1	1.8	4.8	-1.9	-3.9	0.3	1.7

5b. Kimura - Chikuni.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	-7.0	-15.5	-18.4	-5.6	-5.6	-4.9	-16.6	-9.3	-9.3
2	4.1	16.7	20.7	6.0	7.1	5.5	20.5	8.3	14.0
3	-6.7	-9.7	-9.5	-1.9	-1.2	0.4	-7.6	3.6	-7.0
4	17.7	15.6	11.9	1.9	-1.0	-0.3	5.7	-4.2	2.5
5	-8.3	-7.4	-4.9	-0.6	0.4	-0.9	-2.3	1.4	-0.5

5c. SP-Key with true numbers at age and F at age set to 0.6.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Age									
1	-5.7	-14.4	-13.9	-4.3	-3.8	-3.8	-14.6	-7.3	-6.2
2	0.7	11.2	11.6	2.3	2.3	2.7	12.5	5.8	4.8
3	3.3	1.7	1.5	1.7	1.3	0.9	1.7	1.2	1.3
4	2.1	1.1	0.3	0.0	-0.1	-0.1	0.0	0.0	-0.2
5	-0.6	0.2	0.2	0.1	0.1	0.0	0.1	0.1	0.0

Table 6 Results of increasing observation noise for simulated data for cohort slicing, Kimura-Chikuni and SP-Key methods. The error is in units of cv lognormal noise. Performance is reported in terms of the residual sum of squares from ADAPT (RSS) and the mean absolute residual of catch at age and numbers at age (MACR and MANR respectively).

6.a Experiment #1. Parameters as in Table 1.

Method	Slice			K-C			SP-Key			
	RSS	MACR	MANR	RSS	MACR	MANR	RSS	MACR	MANR	
Noise										
0.0	2.26	5.20	35.66	1.45	7.31	17.03	1.24	4.58	20.86	
0.05	2.26	5.31	35.98	1.45	7.72	19.35	1.36	4.77	21.85	
0.10	2.69	5.29	36.65	1.48	8.16	18.52	1.25	4.80	20.93	
0.20	2.04	5.72	32.79	1.50	9.21	19.96	1.28	5.43	21.79	
0.40	2.10	7.38	35.42	1.53	12.38	26.69	1.43	7.38	27.57	
0.60	2.04	9.95	40.05	1.85	15.50	39.96	1.59	10.41	36.75	

6.b Experiment #2. As above except the 4 year old mean is at 90 instead of 85.

Method	Slice			K-C			SP-Key			
	RSS	MACR	MANR	RSS	MACR	MANR	RSS	MACR	MANR	
Noise										
0.0	2.04	4.99	31.27	1.60	7.79	24.19	1.21	4.25	20.16	
0.05	1.54	5.15	23.35	1.67	7.85	25.25	1.22	4.39	20.07	
0.10	1.93	5.10	29.18	1.57	7.95	22.80	1.21	4.49	20.23	
0.20	1.90	5.50	30.60	1.53	8.80	22.94	1.29	5.15	21.13	
0.40	1.84	7.11	29.34	1.79	11.36	32.70	1.34	7.15	25.34	
0.60	1.93	9.64	34.96	1.93	14.87	48.34	1.52	10.11	34.85	

6.c Experiment #3. As Experiment #2 except selectivity is 0.05, 0.75, 1 1 1.

Method	Slice			K-C			SP-Key			
	RSS	MACR	MANR	RSS	MACR	MANR	RSS	MACR	MANR	
Noise										
0.0	1.62	4.99	33.77	1.58	7.79	23.90	1.21	4.71	16.14	
0.05	1.90	5.15	40.90	1.65	7.85	25.03	1.21	4.83	16.02	
0.10	1.88	5.10	42.07	1.56	7.95	22.51	1.21	5.01	16.53	
0.20	2.00	5.50	46.32	1.51	8.80	22.42	1.29	5.73	18.94	
0.40	1.77	7.11	39.29	1.57	11.36	28.34	1.41	7.88	25.67	
0.60	1.83	9.64	45.75	1.95	14.87	49.00	1.58	10.84	38.67	

6.d Experiment #4. As Experiment #3 except twice as many size classes used.

Method	Slice			K-C			SP-Key			
	RSS	MACR	MANR	RSS	MACR	MANR	RSS	MACR	MANR	
Noise										
0.05	2.84	3.84	112.62	1.35	6.90	17.34	1.20	4.72	13.30	
0.2	1.72	4.30	44.04	1.33	6.81	16.32	1.25	5.50	15.92	
0.4	1.59	5.82	36.61	1.47	8.39	27.99	1.20	7.36	19.91	

Table 7. Results of increasing observation noise for simulated data for age based analysis. The error is in units of cv lognormal noise. Performance is reported in terms of the residual sum of squares from ADAPT (RSS) and the mean absolute residual of catch at age and numbers at age (MACR and MANR respectively).

Noise	RSS			MACR		MANR*	
0.00	1.48	0.00	6.03				
0.05	1.50	0.77	7.91				
0.10	1.74	1.55	17.12	15.98			
0.20	2.21	3.12	18.95	14.98			
0.40	2.26	6.47	22.90	27.34			
0.60	2.08	10.36	40.36	40.91			

\* results from a second random seed.

Figure 1. Sample beta distributions

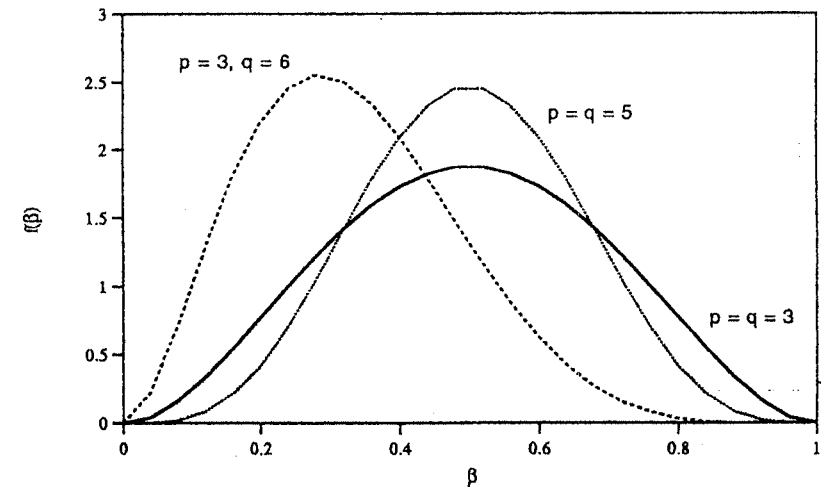




Figure 2. Schematic of SP - Key

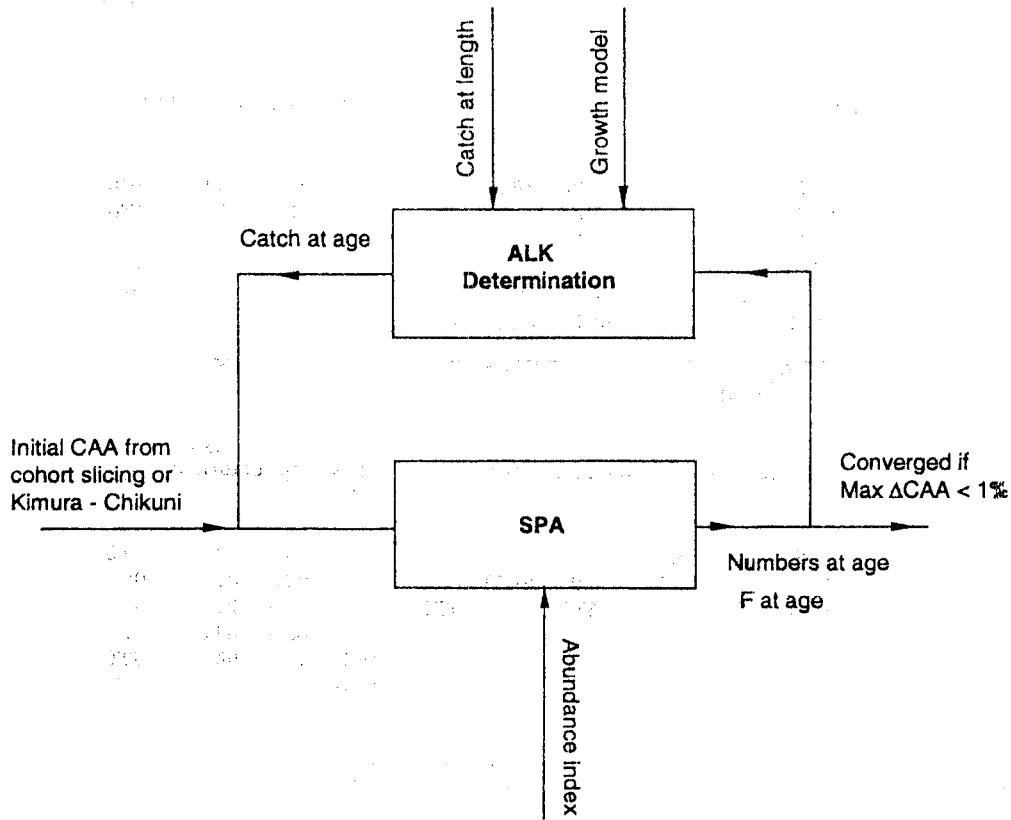
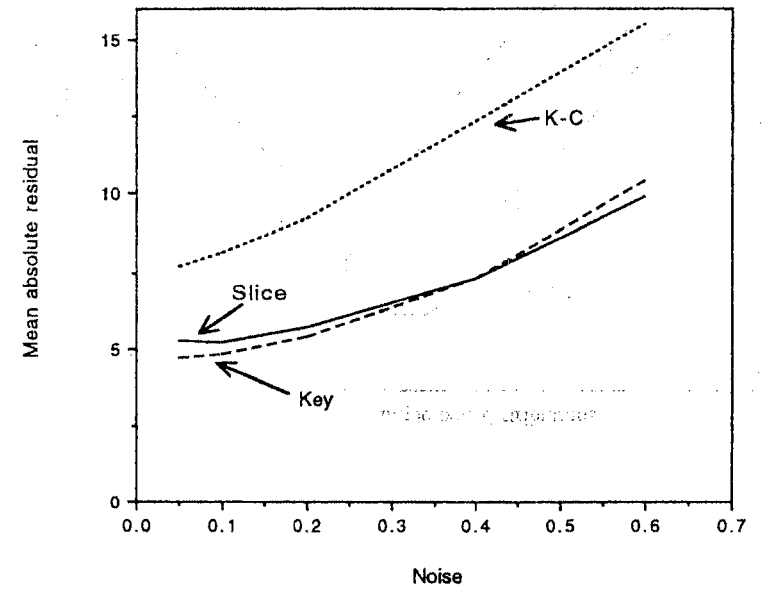


Figure 3. Catch at ages residuals from Experiment 1.



Numbers at age

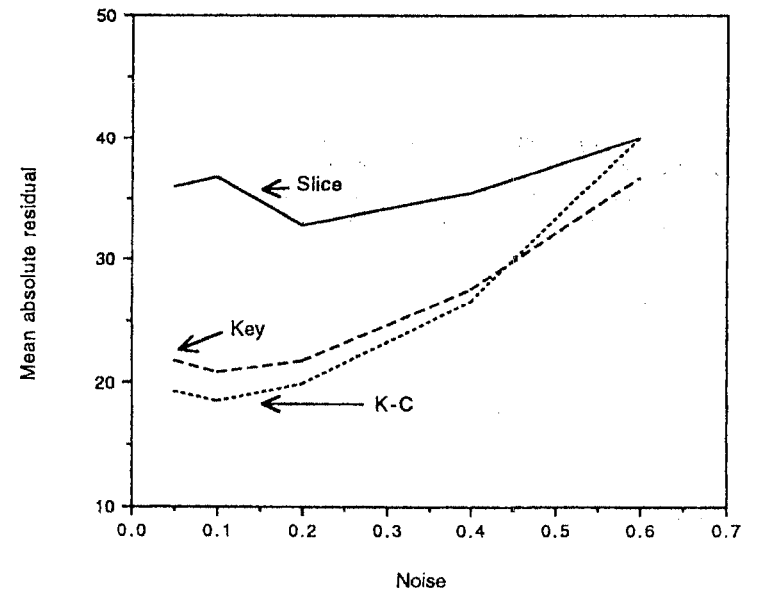
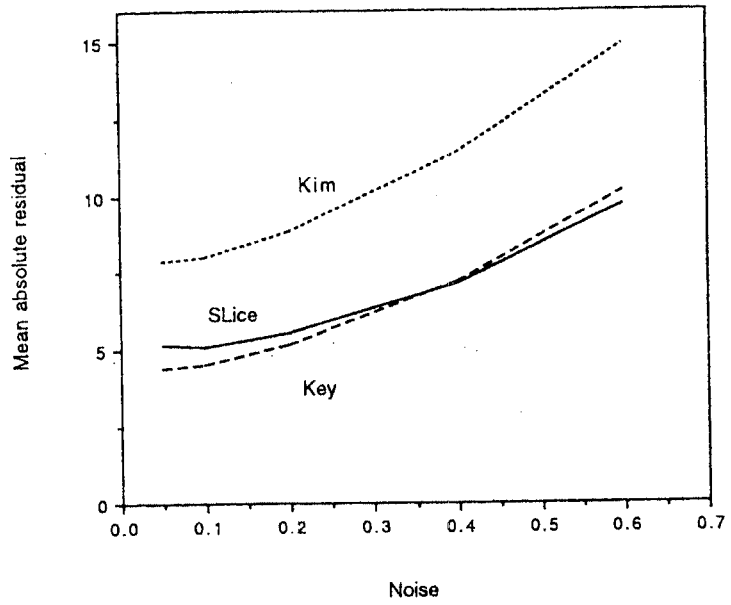


Figure 4. Catch at age residuals from Experiment 2



Numbers at age

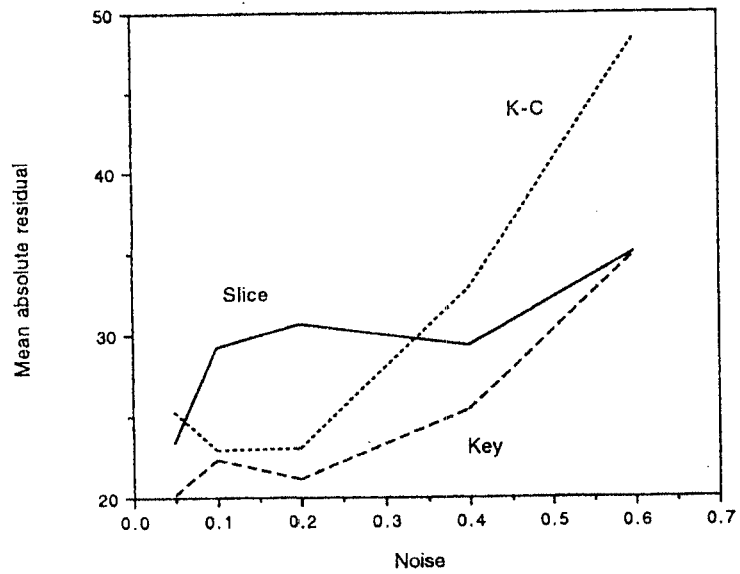
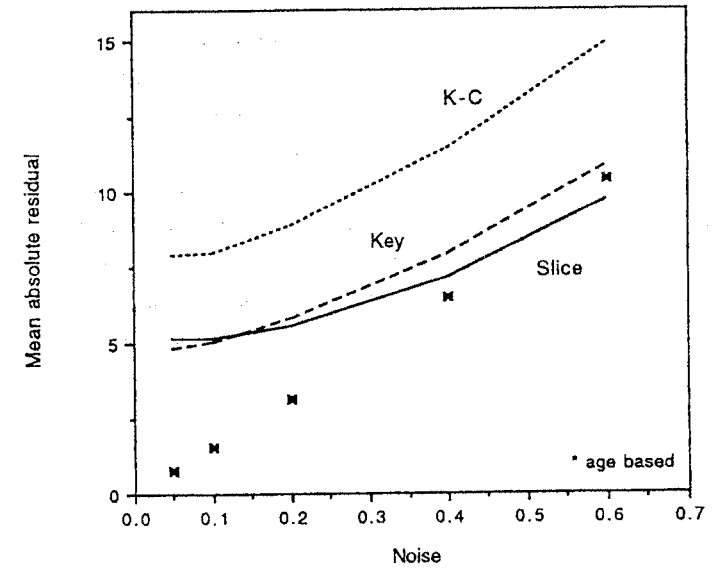


Figure 5. Catch at age residuals from Experiment 3.



Numbers at age

