

BIAS-CORRECTED LENGTH-WEIGHT RELATIONSHIPS FOR ATLANTIC BLUE MARLIN, WHITE MARLIN, AND SAILFISH

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SUMMARY

We present length-weight and weight-length predictive regression equations for Atlantic blue marlin *Makaira nigricans*, white marlin *Tetrapturus albidus*, and sailfish *Istiophorus platypterus*. These equations are intended to allow either lower-jaw fork length or round weight to be predicted from the other. These results are based upon the same data as used in earlier studies, but contain two refinements. First, the equations have been corrected slightly to account for the bias introduced by log transformation. Second, we present—in addition to equations for each sex separately—analyses of both sexes combined for each species, so that unbiased conversions can be made even when the sex of the animal is not known. The method of bias correction illustrated here can be employed whenever power functions are fit by regression after log transformation.

RESUME

Nous présentons les équations de régression prédictive de longueur-poids et poids-longueur du makaire bleu de l'Atlantique *Makaira nigricans*, makaire blanc *Tetrapturus albidus* et voilier *Istiophorus platypterus*. Ces équations servent à prédire la longueur fourche à la mâchoire inférieure et le poids vif à partir d'un d'entre eux. Ces résultats sont basés sur les mêmes données utilisées dans des études antérieures, mais contiennent deux affinages. Premièrement, les équations ont été légèrement corrigées pour obtenir le biais introduit par transformation de logarithme. Deuxièmement, nous présentons --en plus des équations pour chaque sexe séparé-- des analyses des deux sexes combinées pour chaque espèce, de façon à ce que les conversions non biaisées puissent être élaborées même si le sexe de l'animal n'est pas connu. La méthode de correction de biais illustrée ici peut être utilisée lorsque les fonctions de puissance sont ajustées par régression après transformation de logarithme.

RESUMEN

Se presentan las ecuaciones de regresión predictivas de talla-peso y peso-talla para la aguja azul atlántica (*Makaira nigricans*), aguja blanca (*Tetrapturus albidus*) y pez vela (*Istiophorus platypterus*). Se intenta que estas ecuaciones permitan predecir la longitud horquilla-mandíbula inferior o peso vivo a partir de otras. Estos resultados se basan en los mismos que se utilizaron en estudios anteriores, pero contienen dos mejoras. En primer lugar, las ecuaciones han sido ligeramente modificadas para explicar el sesgo introducido por transformación de los cuadernos de pesca. En segundo, presentamos, además de las ecuaciones para cada sexo por separado, análisis de ambos sexos combinados para cada especie, de manera que puedan hacerse las conversiones sin sesgo a pesar de no conocer el sexo del animal. El método de corrección de sesgo que se presenta puede emplearse siempre que las funciones de potencia se ajusten mediante la regresión tras la transformación de los cuadernos de pesca.

INTRODUCTION

Prince and Lee (1989) and Lee and Prince (1990) presented a detailed analysis of length and weight relationships for the Atlantic blue marlin *Makaira nigricans*, white marlin *Tetrapturus albidus*, and sailfish *Istiophorus platypterus*. In particular, they presented equations for estimating weight from length (total length or lower-jaw fork length) and length from weight. Here, we reanalyze much of the same data to arrive at slightly revised equations. The refinements added are corrections of the predictive equations for the bias due to log transformation, and the addition of equations derived from data on both sexes and intended for use when the sex of a fish is not known.

In fitting allometric equations to sample data, it is usually convenient to use a log transform. For example, instead of fitting directly a length weight relationship of the form

$$(1) \quad w = a \cdot l^b,$$

where w is the weight of a fish and l its length, one can take the natural logarithm of each side of the equation so that it becomes

$$(2) \quad \ln(w) = \ln(a) + b \cdot \ln(l).$$

This transformation allows estimating the constants a and b in the above equation by ordinary least squares (OLS). In addition, the transformation is theoretically consistent with the observation that the coefficient of variation of length at weight (or of weight at length) tends to be constant.

The logarithmic transformation, however, introduces a systematic bias into the resulting equation when it is used for prediction. The correction for this bias was reported in the ecological literature by Baskerville (1972) and Whitaker and Marks (1975); a minor correction to those results was given by Sprugel (1983). The correction is not complex. If we call a prediction from equation (2) \hat{Y}_{OLS} and refer to the correction factor as CF , then the corrected prediction is

$$(3) \quad \hat{Y}_{corr} = \hat{Y}_{OLS} \cdot CF.$$

The correction factor as given by Sprugel (1983) is

$$(4) \quad CF = \exp\left(\frac{SSE}{2(n-2)}\right),$$

where SSE is the sum of squared errors from the OLS regression [equation (2)] and n is the sample size.

APPLICATION

Analyses were performed for the most common measure of length, lower-jaw fork length (LJFL). Because sample sizes were quite large, the correction factors were within 1% of unity. Nevertheless, it is desirable to correct for a known bias. It should be remembered, moreover, that the correction factor can be much larger when analyzing smaller data sets, such as those used for other length-weight relationships described by Prince and Lee (1989) and Lee and Prince (1990) for the Istiophoridae.

For predicting round weight from lower-jaw fork length, we use the notation

$$(5) \quad \hat{w} = a \cdot l^b.$$

For predicting lower-jaw fork length from round weight, the corresponding notation is

$$(6) \quad \hat{l} = c \cdot w^d.$$

The bias-corrected coefficients a , b , c , and d of equations (5) and (6) are tabulated in Table 1.

COMMENT

The method of bias correction illustrated here is applicable to allometric equations, fecundity relationships, and other log-transformed power relationships. The computation and application of the correction factor are straightforward, and it can be combined with the regression coefficient to avoid extra computation in applying the resulting predictive equation. An example of a relatively large correction factor is given by Saila *et al.* (1988, p. 149) in an age-fecundity relationship for yellowtail flounder. That example demonstrates that the correction factor encountered in actual fisheries applications can be much larger than the correction factors obtained for the equations given here.

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Table 1. Coefficients of bias-corrected equations for predicting round weight (kg) from lower-jaw fork length (cm) using the equation $w = a \cdot l^b$ or predicting lower-jaw fork length from round weight using the equation $l = c \cdot w^d$. Bias-correction factors have been incorporated into parameters a and c so that no further correction is necessary. For explanation, see text.

Sex	Sample size (n)	Weight range, kg	Length range, cm	Coefficients			
				a	b	c	d
Blue Marlin							
♀	3267	0.06-540.9	23.0-378.5	1.9034×10^{-6}	3.2842	61.731	0.28180
♂	1978	0.06-178.0	23.0-277.0	2.4682×10^{-6}	3.2243	61.961	0.28137
♀ ♂	5245	0.06-540.9	23.0-378.5	1.1955×10^{-6}	3.3663	62.010	0.28065
White Marlin							
♀	3149	2.7-67.1	91.4-205.0	3.9045×10^{-6}	3.0694	78.423	0.23191
♂	1719	3.6-41.3	96.0-195.5	1.9556×10^{-5}	2.7487	76.847	0.23548
♀ ♂	4868	2.7-67.1	91.4-25.0	5.2068×10^{-6}	3.0120	76.460	0.23888
Sailfish							
♀	1280	0.04-52.7	27.1-204.5	1.1441×10^{-6}	3.2683	74.614	0.26460
♂	907	0.04-30.1	27.1-188.0	1.6922×10^{-6}	3.1879	70.907	0.28191
♀ ♂	2187	0.04-52.7	27.1-204.5	1.2869×10^{-6}	3.2439	72.962	0.27201