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CASTING THE SHEPHERD STOCK-PRODUCTION MODEL IN A STATISTICAL FRAMEWORK SUITABLE FOR SWORDFISH STOCK ASSESSMENT AND MANAGEMENT ADVICE

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SUMMARY

A non-equilibrium stock-production model proposed by Shepherd (1987) is suggested as a model that may be useful for Atlantic swordfish stock assessments and management advice. The model is simple, flexible, and is parameterized in a manner that allows for straightforward interpretation of the parameters in a fishery/population dynamics context. However, as proposed by Shepherd, the model lacks a formal statistical basis. A statistical framework for Shepherd's model is developed herein. The statistical model allows both measurement and process error, variance estimates of all parameters of interest, and diagnostics useful for judging the adequacy of the results.

RESUME

Un modèle de production de stock ne postulant pas de conditions d'équilibre, proposé par Shepherd (1987) est suggéré comme modèle pouvant s'avérer utile pour l'évaluation de stock et les avis de gestion concernant l'espadon de l'Atlantique. Ce modèle est simple, flexible, et est paramétrisé de façon à permettre une interprétation directe des paramètres dans un contexte pêcherie/dynamique des populations. Cependant, tel qu'il est proposé par Shepherd, le modèle manque de base statistique formelle. Une structure statistique est donc élaborée ici pour le modèle de Shepherd. Le modèle statistique tient compte des erreurs de mensuration comme de traitement, permet d'estimer la variance de tous les paramètres intéressants, et de formuler des diagnostics utiles pour jauger l'exactitude des résultats.

RESUMEN

Se sugiere que el modelo de producción en condiciones de no equilibrio propuesto por Shepherd (1987) podría resultar útil en las evaluaciones del stock de pez espada atlántico y a efectos de gestión. El modelo es sencillo, flexible y permite una interpretación directa de los parámetros en el contexto de una dinámica pesquería/población. Sin embargo, y de acuerdo con Shepherd, el modelo no tiene la suficiente base estadística. En el presente documento se establece un marco estadístico para el modelo de Shepherd. El modelo estadístico permite medir y procesar el error, estimaciones de varianza de todos los parámetros de interés y hacer diagnósticos útiles para juzgar si los resultados son adecuados.

INTRODUCTION

Renewed interest in the use of stock-production models for assessment of the status of stocks of Atlantic swordfish has surfaced during the past year (Vaughn et al. 1991; Fonteneau 1991). Preliminary stock-production analyses were explored when ICCAT first addressed the status of swordfish stocks in the early 1980's (Kikawa and Honma 1981; Farber and Conser 1983). Stock-production models were also used for assessment of Pacific swordfish during the same period (Sakagawa and Bell 1980). However, difficulties were encountered in finding adequate indices of abundance and in fitting the models with available data. Further the equilibrium models that were employed juxtaposed against a highly dynamic swordfish fishery (among other difficulties), gave results that were less than ideal for fishery management purposes in the ICCAT arena. Age-structured models were first used for preliminary assessment work in the mid-1980's (Conser et al. 1985), and thereafter interest in stock-production models waned.

In retrospect the demise of all further exploration of stock-production models for swordfish (in favor of age-structured models) may have been premature for several reasons:

(1) Estimation of growth rates for older swordfish (say, ages 10+) has been and continues to be problematic using the available hard part studies and/or mark-recapture data.

(2) The development of age-length keys has not been possible due to problems in age determination coupled with the appreciable logistic problems involved in sampling annually throughout the wide geographical range of Atlantic swordfish. Consequently cohort slicing has been employed to estimate catch-at-age from size frequency samples and landings data.

(3) Available sex ratio data (Hoey and Mejuto 1991) indicate that large swordfish tend to be females. However, the causal mechanism for this phenomenon has not been established. Credible hypotheses include sexually dimorphic growth, differences in natural mortality by sex, and differences in availability by sex; or some combination of one or more of the above. These apparent differences in population vital rates by sex further exacerbate difficulties with the age-structured models. Depending upon the actual causal mechanism, these differences cause problems in estimating catch-at-age from the size samples and landings data (neither of which are available by sex) and/or complicate the subsequent virtual population analysis (VPA).

(4) The development of non-equilibrium stock-production models (Fletcher 1978; Rivard and Bledsoe 1978; Shepherd 1987; Schnute 1989) has made it possible to relax the tenuous equilibrium assumption commonly required with most traditional stock-production models (e.g. those of Schaefer 1954; Fox 1975, etc.).

(5) Other stock-production model assumptions that are tenuous for other stocks, are not unreasonable for swordfish, e.g.
   (a) constant exploitation pattern due to the predominance of a single gear type (i.e. longline) since the early 1960's; and
   (b) continuous recruitment throughout the year due to a protracted spawning season.

The above notwithstanding, practical problems remain in using stock-production models for assessing swordfish stocks and for providing management advice. The general problem of attaining viable parameter estimates given a data-limited scenario is troublesome. In particular for Atlantic swordfish, the short duration of currently available indices of abundance from directed fisheries (1981 to present) is a serious limitation (Vaughn et al. 1991).

The primary purpose of this paper is to propose a non-equilibrium stock-production model that may be useful for swordfish assessment and management advice. The non-equilibrium stock-production model of Shepherd (1987) is suggested. It is a simple and flexible model, parameterized such that all parameters of interest are meaningful and interpretable in a fishery/population dynamics context. However, as proposed by Shepherd (1987), it lacks a formal statistical basis. A statistical framework for Shepherd's model is developed herein. The statistical model allows both measurement and process error, variance estimates of all parameters of interest, and diagnostics useful for judging the adequacy of the results.

Statistical Model for Stock-Production

THE SHEPHERD STOCK-PRODUCTION MODEL

Define terms as follows:

B_y exploited stock biomass at the beginning of year y

P_y net production during year y; growth in stock biomass due to recruitment and individual fish growth minus losses due to natural mortality

C_y catch (in weight) taken during year y

Then the first order difference equation

$$B_{y+1} = B_y + P_y - C_y \tag{1}$$

relates the exploited stock biomass at the beginning of a year, B_{y+1}, to the exploited stock biomass at the beginning of the previous year, B_y, plus net production, P_y, minus the catch in weight, C_y.

A curvilinear relationship is assumed between the production-biomass ratio, P_y/B_y, and exploited biomass, B_y. The *first form* of the Beverton and Holt (1956) stock-recruitment relationship is used to model positive production, i.e. growth rate due to both recruitment and somatic growth of individuals. Natural mortality is then expressed explicitly, giving the production-biomass ratio:

$$\frac{P_y}{B_y} = \frac{a}{1 + \frac{B_y}{K}} - M \tag{2}$$

where

a parameter of the Beverton-Holt relationship; in this application represents the maximum instantaneous growth rate of positive production; has dimension yr^{-1}

K parameter of the Beverton-Holt relationship; represents the threshold stock biomass above which density dependent effects dominate; has dimension of biomass

M instantaneous rate of natural mortality; assumed constant over years; has dimension yr^{-1}

Reparameterization is accomplished by letting $a = (\alpha' + 1)M$. It then follows that $K = B_{max}/\alpha'$. Substituting into Equation 2 and simplifying gives production as a function of biomass:

$$P_y = \alpha' M B_y \left(\frac{1 - \frac{B_y}{B_{max}}}{1 + \frac{\alpha' B_y}{B_{max}}} \right) \tag{3}$$

where

α' is a unitless measure of the resilience of the stock; $M\alpha'$ is the slope of the stock-production curve (i.e. Equation 3) at the origin; Shepherd indicates that information from stocks where *good* data exist suggests that α' is likely to be in the range 1 to 10

B_{max} is the exploitable virgin stock biomass, i.e. the portion of the virgin biomass that could have been exploited using the constant exploitation pattern observed after the fishery was initiated

The stock-production relationship has three parameters, i.e. α', M, and B_{max}. Solving Equation 3 for B_{max} gives

$$B_{max} = B\alpha' \left(\frac{\frac{P}{B} + M}{\alpha' M - \frac{P}{B}} \right) \tag{4}$$

where for a stock-production curve defined by α' and M, any ordered pair (B,P) that lies on the curve can be used to calculate the value of B_{max}. Shepherd suggests that in practice B_{max} can often be approximated by assuming that the curve passes through the joint mean $(\bar{B}, \bar{P})$, where the joint mean is calculated from the observed B_i and P_i. Note that Equation 4 differs slightly from the corresponding equation in Shepherd (1987), which contained a typographical error.

Differentiating Equation 3 with respect to biomass, setting $dP/dB = 0$, and solving for B gives the stock biomass where maximum net production will occur:

$$B_{msy} = \left(\frac{\sqrt{1+\alpha'} - 1}{\alpha'} \right) B_{max} \tag{5}$$

Substituting B_{msy} into Equation 3 gives the maximum sustainable yield (MSY):

$$MSY = \frac{\alpha' M}{\sqrt{1+\alpha'}} B_{msy} \left(1 - \frac{B_{msy}}{B_{max}} \right) \tag{6}$$

Note that Equation 6 differs from the corresponding equation in Shepherd (1987), which contained typographical errors.

STATISTICAL MODEL FOR STOCK-PRODUCTION

Using Shepherd's stock-production relationship (i.e. Equation 3), a statistical model for parameter estimation is proposed as follows.

Indices of abundance for the exploited stock biomass (b_y), indices of net production (p_y), and an index of virgin biomass (b_{max}) are given by

$$\begin{aligned} b_y &= qB_y \\ p_y &= qP_y \qquad \textit{for } y=1,\dots,Y \\ b_{max} &= qB_{max} \end{aligned} \tag{7}$$

where

q catchability of the unit of standardized effort associated with the index of stock biomass, b_y

Y number of years of available data

Multiplying both sides of Equation 1 by q and introducing a process error term, gives

$$b_{y+1} = \left(b_y + p_y - qC_y \right) e^{\epsilon_y} \tag{8}$$

where ϵ_y is a normally distributed random variable with mean 0 and variance σ_ϵ^2 representing the process error and p_y is obtained by multiplying both sides of Equation 3 by q, i.e.

$$p_y = \alpha' M b_y \left(\frac{1 - \dfrac{b_y}{b_{max}}}{1 + \dfrac{\alpha' b_y}{b_{max}}} \right) \tag{9}$$

The measured index of stock biomass (b'_y) is related to the *true* index of abundance by

$$b'_y = b_y e^{\eta_y} \qquad \textit{for } y=1,\dots,Y \tag{10}$$

Similarly the measured catch in weight (C'_y) is related to the *true* catch by

$$C'_y = C_y e^{\delta_y} \qquad \textit{for } y=1,\dots,Y-1 \tag{11}$$

where η_y and δ_y are normally distributed random variables, which represent the measurement errors in the index of exploited biomass and in catch weight, respectively.

Statistical Model for Stock-Production

The objective function to be minimized is

$$\lambda_e \sum_{y=1}^{Y-1} e_y^2 + \sum_{y=1}^{Y} \eta_y^2 + \lambda_\delta \sum_{y=1}^{Y-1} \delta_y^2 \qquad (12)$$

where λ_e and λ_δ are relative weights for the process error and catch measurement error, respectively (relative to the measurement error for index of exploited biomass).

The complete list of potential model parameters is:

- b_y — *true* index of exploited biomass; $y=1,...,Y$
- C_y — *true* catch in weight; $y=1,...,Y-1$
- q — catchability; 1 parameter
- α' — stock resiliency; 1 parameter
- b_{max} — index of virgin biomass; 1 parameter
- M — natural mortality; 1 parameter

In practice, M is rarely estimable because it is highly correlated with q. It is generally fixed at some appropriate level, as is commonly done in yield-per-recruit or virtual population analyses (e.g. $M=0.2\ yr^{-1}$).

Further if the error in estimating catch weight (δ_y) is small relative to measurement error in the index of exploited biomass (η_y) and the process error (ϵ_y), then it will have little impact and can be safely ignored. This eliminates the need to estimate the C_y and simplifies the objective function to be minimized, i.e.

$$\lambda_e \sum_{y=1}^{Y-1} e_y^2 + \sum_{y=1}^{Y} \eta_y^2 \qquad (13)$$

Statistical Model for Stock-Production

Generally most of the process error (ϵ_y) will be attributable to Equation 9 since stock-production data are notoriously noisy. However, placing the process error term in Equation 8 provides more generality. If catch measurement error (δ_y) is ignored or if catchability (q) is fixed using auxiliary information, then all error due to these factors will be incorporated into the process error term.

The process error is assumed to be lognormally distributed mainly to provide stability in the multi-criteria objective function (Equation 12 or 13). In general the distribution of the process error is difficult to know *a priori*. However, it should be noted that assuming normally distributed process error in this model makes the model a Kalman filter, which has some theoretical advantages. However, it makes the problem of selecting the appropriate weighting factor (λ_e) much more difficult.

The b_y are generally estimable in the neighborhood of the b'_y. However, due to the limited data used in stock-production models (i.e. total catch and an index of biomass), the remaining parameters (q, α', and b_{max}) will generally not be estimable simultaneously. Generally two of these can be estimated by fixing the third (in addition to fixing M). However, when α' is fixed, q and b_{max} may be correlated for some levels of α'. In all cases, the correlation matrix of estimated parameters must be examined carefully.

Using the full statistical model (Equation 12) and assuming that M and one of the triplet (q, α', b_{max}) are fixed, we have

3Y-2	residual error terms
Y+(Y-1)+2	parameters to be estimated
Y-3	degrees of freedom

If errors in catch weight are negligible and we use the reduced statistical model (Equation 13), we have

2Y-1	residual error terms
Y+2	parameters to be estimated
Y-3	degrees of freedom

The most satisfactory approach, in general, will be depend on the data at hand, as well as on auxiliary data or other information that may help in judiciously fixing some subset of the parameter triplet (q, α', b_{max}). However, the usual tact will be to use the reduced model (Equation 13); to fix b_{max} (and M); and to estimate q and α'.

In some cases, b_{max} may be fixed using indices of exploited biomass from years shortly after a fishery was initiated (e.g. Japanese longline indices for tunas from the mid-1950's). This will not always be practical, however, due to difficulties in standardizing data from the early years of a fishery or simply due to the absence of data from that period. An alternative, analogous to Shepherd's (1987) use of Equation 4, can be derived by multiplying both sides of Equation 4 by q, and assuming the stock-production curve (Equation 9) passes through the joint mean $(\bar{b},\bar{p})$, giving

$$b_{max} = \bar{b}\alpha' \left(\frac{\frac{\bar{p}}{\bar{b}} + M}{\alpha' M - \frac{\bar{p}}{\bar{b}}} \right) \quad (14)$$

Then for fixed (q,α'), b_{max} is approximated from Equation 14 using the observed b'_y (ignoring measurement error) to estimate $\bar{b}$ and using the p_y from Equation 8 (ignoring process error) to estimate $\bar{p}$.

It may also be possible to fix α' using knowledge of the life history characteristics of the species of interest. Generally long-lived, highly fecund species with many ages in the spawning stock should have a higher resiliency (α') than short lived species. Such a comparative approach may make it possible to isolate α' to a few discrete values (within the range 1-10), rather than making it a parameter to be estimated.

Since stock-production models are often over-parameterized, every attempt should be made to make the model as parsimonious as possible.

After all auxiliary data and other information have been used to reduce the estimable parameter set as much as possible, the remaining parameters are estimated by minimizing Equation 13 (or Equation 12) using nonlinear least squares and a Marquardt algorithm. Initial values (for all possible model parameters) may be derived from:

b_y Observed values (b'_y)

C_y Observed values (C'_y)

q VPA results, if available
b/B from some year when estimates of b and B are available

α' Examination of life history characteristics
Examination of recruitment patterns

b_{max} Indices from early years of the fishery, if available
Application of Equation 14

Diagnostics provided by the model include:

(a) Marquardt performance results, e.g. convergence criteria, orthogonal offset at the solution, final sum of squares, etc.

(b) Mean square error (MSE) of the overall fit

(c) Standard error and c.v. for each estimated parameter (assuming linearity at the optimal solution)

(d) Correlation among estimated parameters

(e) Standardized residuals for measurement and process errors separately

(f) Percent of the total sum of squares at the solution attributable to each residual error term; with sums for process and measurement error

Once acceptable parameter estimates have been achieved, the management criteria B_{msy} and MSY can be calculated from Equations 5 and 6, respectively. Variance estimates of B_{msy} and MSY (as well as all other state variables in the model) are available via bootstrapping.

DISCUSSION

The Shepherd stock-production model has been exercised with good results on several North Sea stocks and on simulated data (Shepherd 1987; ICES 1987). The statistical framework for the model, described in this paper, has previously been employed in conjunction with several different assessment methods. It has been used as a framework for DeLury models (Collie and Sissenwine 1983; Conser 1991); and for stock-recruitment analysis (Walters and Ludwig 1981). The combination of the simple structure of the Shepherd stock-production model and the flexibility and diagnostic features of the statistical model make for a versatile stock assessment tool. These characteristics are especially important when dealing with a limited data scenario.

Provided an adequate index of stock biomass can be developed, covering a reasonable series of years (perhaps 10 to 15 years), the approach may prove quite useful for swordfish assessment and for providing management advice.

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