Col.Vol.Sci.Pap. ICCAT, 39 (2) : 497-502 (1992)

POSSIBLE BIASES IN THE VPA ESTIMATES OF POPULATION SIZES OF THE PLUS GROUP

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SUMMARY

Biases in estimating population sizes of the plus group are examined. It is shown that the VPA estimates used in the 1990 ICCAT assessment are sensitive to the systematic ageing errors. The biases in the estimates of population sizes of the plus group might increase exponentially with tracing back to the earlier year.

RESUME

Les biais de l'estimation de la taille de la population du groupe plus sont examinés. Il est démontré que les estimations de VPA qui utilisent l'évaluation de 1990 de l'ICCAT sont sensible aux erreurs systématiques de la détermination de l'âge. Les biais de l'estimation de la taille de la population du groupe plus pourraient s'accroître de façon exponentielle en revenant vers les premières années.

RESUMEN

Se estudian los sesgos en la estimación de tamaños de la población del grupo plus. Se observa que las estimaciones del VPA usadas en la evaluación de ICCAT en 1990 eran sensibles a los errores sistemáticos de determinación de la edad. Los sesgos en las estimaciones de tamaños de la población del grupo plus podrían aumentar de forma exponencial remontándose hacia años anteriores.

INTRODUCTION

The ICCAT assessments for bluefin tuna and swordfish are conducted with virtual population analysis(VPA) using catch-at-age data. Since the larger fish cannot be assigned to ages well, they are pooled into a plus group. Population numbers in the plus group at year y are assumed to be survivors from the plus group at year y-1 and survivors from the year class one year younger than the plus group at year y-1.

In the 1990 ICCAT assessment (ICCAT,1991), bluefin tuna and swordfish were grouped at ages 10 and 5, respectively and the population sizes of the plus group were estimated by the alpha method.

In this paper, the possibele biases in the estimates of population sizes of the plus group using the alpha method are examined.

METHODS AND AN EXAMPLE

METHODS

Let y=index for year, t=youngest age present in the plus group. The alpha method assumes that the fishing mortality of the last two ages are related by a proportion α : $F_{t+,y-1} = \alpha F_{t-1,y-1}$. The population sizes of the plus group N_{t+} are estimated by backwards calculation using the survival equation and the catch equations

$$N_{t+,y} = N_{t-1,y-1}\exp(-F_{t-1,y-1} - M) + N_{t+,y-1}\exp(-\alpha F_{t-1,y-1} - M) \quad (1)$$

$$C_{t-1,y-1} = \frac{F_{t-1,y-1}}{F_{t-1,y-1}+M} N_{t-1,y-1}\{1 - \exp(-F_{t-1,y-1} - M)\} \quad (2)$$

$$C_{t+,y-1} = \frac{\alpha F_{t-1,y-1}}{\alpha F_{t-1,y-1}+M} N_{t+,y-1}\{1 - \exp(-\alpha F_{t-1,y-1} - M)\}. \quad (3)$$

In the 1990 ICCAT assessment, α is assumed to be 1 for bluefin tuna and 1.116 for swordfish.

Using Pope's approximation (Pope,1972), $N_{t+,y-1}$ and $N_{t-1,y-1}$ are given by (see Appendix A for derivation)

$$N_{t+,y-1}=\frac{C_{t+,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}N_{t+,y}\exp(M)$$

$$+C_{t+,y-1}\frac{C_{t+,y-1}+C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}\exp(M/2) \qquad (4)$$

$$N_{t-1,y-1}=\frac{\beta_{y-1}C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}N_{t+,y}\exp(M)$$

$$+\beta_{y-1}C_{t-1,y-1}\frac{C_{t+,y-1}+C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}\exp(M/2) \qquad (5)$$

where $\beta_{y-1}=1+(1-F_{t-1,y-1}/2)(\alpha-1)$.

When $\alpha=1$, $N_{t+,y-1}$, and $N_{t-1,y-1}$ can be represented by explicit function of $N_{t+,y}$, $C_{t+,y-1}$, and $C_{t-1,y-1}$.

$$N_{t+,y-1}=\frac{C_{t+,y-1}}{C_{t+,y-1}+C_{t-1,y-1}}N_{t+,y}\exp(M)+C_{t+,y-1}\exp(M/2) \qquad (6)$$

$$N_{t-1,y-1}=\frac{C_{t-1,y-1}}{C_{t+,y-1}+C_{t-1,y-1}}N_{t+,y}\exp(M)+C_{t-1,y-1}\exp(M/2) \qquad (7)$$

Those equations indicate that for the precise estimation, the ratio of $C_{t+,y-1}$ and $C_{t-1,y-1}$ as well as the values of $C_{t+,y-1}$ and $C_{t-1,y-1}$ are important.

Estimator of stock sizes of the plus group using the alpha method is similar to the VPA stock size estimator. N_{t+} for the earlier years are determined uniquely by $N_{t+,y}$ and catch-at-age data. The difference arises from the appearance of the ratio of catches in the first term of the right-hand side of the equations.

In order to examine the effect of ageing errors on the stock size estimates, we assume that due to the systematic ageing errors, $C_{t+}+\delta$ and $C_{t-1}-\delta$ are obtained instead of the true values C_{t+} and C_{t-1}. For simplicity, we assume $\alpha=1$ in the following analyses. Let $\overline{N}_{t+}$ be the true value of N_{t+}. The estimate of N_{t+} is given by

$$N_{t+,y-1}=\frac{C_{t+,y-1}+\delta}{C_{t+,y-1}+C_{t-1,y-1}}\overline{N}_{t+,y}\exp(M)+(C_{t+,y-1}+\delta)\exp(M/2)$$

$$=\overline{N}_{t+,y-1}+\frac{\delta}{C_{t+}+C_{t-1}}\overline{N}_{t+}\exp(M)+\delta\exp(M/2). \qquad (8)$$

Assuming that $\delta=\varepsilon C_{t+}$, equation (8) becomes

$$N_{t+,y-1}=(1+\varepsilon)\overline{N}_{t+,y-1}. \qquad (9)$$

Assuming that there are same ageing errors in all years and F is constant from year to year, the estimate of $N_{t+,y-j}$ is given by (See Appendix B)

$$N_{t+,y-j}=(1+\varepsilon)\left(1+\rho\varepsilon\frac{1-\{\rho(1+\varepsilon)\}^{j-1}}{1-\rho(1+\varepsilon)}\right)\overline{N}_{t+,y-j} \qquad (10)$$

where,

$$\rho=1-\frac{F}{F+M}(1-\exp(-F-M))\exp(M/2). \qquad (11)$$

In the limit of $\rho\rightarrow 1$, equation (10) becomes

$$N_{t+,y-j}=(1+\varepsilon)^{j}\overline{N}_{t+,y-j} \qquad (12)$$

EXAMPLE

Restrepo and Powers (1991) studied the effects of systematic ageing errors on the VPA estimates. Table 1c of Restrepo and Powers (1991) shows true catch-at-age matrix of the hypothetical population which are loosely based on the North Atlantic swordfish data. They assumed that the lengths of fish in the true population were normally distributed around each age. True catch-at-age matrix are converted into length groups and they were reassignd to age group (age slicing). Table 3a of Restrepo and Powers (1991) shows incorrect catch-at-age matrix resulting misassignments of ages to the catches.

The values of C_{14} and C_{15+} are changed remarkably and the values of $C_{15+,y-1}/(C_{14,y-1}+C_{15+,y-1})$ are also changed from about 0.5 to 0.9.

RESULTS AND DISCUSSION

Equation (12) indicates that in some cases, the biases in the estimates of population sizes of the plus group may increase exponentially with tracing back to the earlier year. On the other hand, the biases in the standard VPA estimates of population sizes do not increase exponentially in the presence of systematic errors in ageing.

In order to evaluate the biases, equation (10) is used. Assuming that $M=F=0.14$, positive biases of 10% and 20% ($\varepsilon=0.1$ and 0.2) in the catch-at-age data cause overestimation of population sizes of the plus group 156% and 758%, respectively, in tracing back to 20 years and -10% and -20% negative biases ($\varepsilon=-0.1$ and -0.2) cause -49% and -69% underestimation, respectively.

Table 1 shows the estimates of N_{15+} and F_{15+} using biased catches (Table 3a of Restrepo and Powers 1991) by the alpha method. The estimate of N_{15+} using biased catch-at-age is largely different from true N_{15+}. The true N_{15+} is reduced to 1/2 during 11 years, while the estiamtes of N_{15+} is reduced to 1/20. This result indicates that the positive biases in N_{15+} increase as tracing back to the earlier year.

Taking account of this, the 1990 ICCAT assessments for bluefin tuna and swordfish are examined.

BLUEFIN TUNA

Table 2 shows the values of $C_{10+}/(C_9+C_{10+})$ for bluefin tuna (data from ICCAT,1991). When this ratio is close to 1.0, the estimates of N_{10+} increase as tracing back to the earlier year. In particular, according to equation (6), when

$$\frac{C_{t+,y}}{C_{t+,y}+C_{t-1,y}} > \exp(-M) \quad , \tag{13}$$

N_{t+} can increase as tracing back to the earlier year even only from the first term of the right-hand side of equation (6). Since M is assumed to be 0.14 for bluefin tuna, $\exp(-M)$ is 0.869. Table 2 shows that the ratio is larger than 0.869 in 11 years. Remarkable decreasing trend in N_{8+} from 1974 to 1980 in the 1990 ICCAT assessment is caused by the large values of this ratio from 1974 to 1980. It shoud be noted that while estimates of N_{8+} is reduced to 1/3 from 1974 to 1980, there is no consistent trends in the CPUE (Suzuki,1985).

Using the fishing mortality F estimated by the 1990 ICCAT assessment (ICCAT,1991), stock size at age (more than 10 years old) in 1990 are evaluated (Table 3). For example, $N_{30+,1990}$ is calculated from the survival numbers of $N_{10+,1970}$ in 1990 as follows:

$$N_{10+,1970}\times\exp\{-\sum_{y=1970}^{1989}(F_{10+,y}+M)\}=234911\times\exp\{-(2.7846+2.8)\}$$
$$=883 \tag{14}$$

This corresponds to 6.6% of $N_{10+,1990}$, which means 6.6% of $N_{10+,1990}$ are more than 30 years old. The life span of bluefin tuna is supposed to be about 30 years old and virtually no $N_{10+,1970}$ fish are anticipated to survive by 1990. Hence, this result is unrealistic and suggests the overestimation of N_{10+} forthe earlier years.

SWORDFISH

Since the catches of the older fish of swordfish were grouped at ages 5, the effects of systematic ageing errors may be neglegible. In the example of Restrepo and Powers (1991), C_4, C_{5+}, and $C_{5+,y-1}/(C_{4,y-1}+C_{5+,y-1})$ are only changed slightly. Sexually dimorphic growth, however, could be a cause of the systematic ageing errors.

CONCLUSION

Since the alpha method is not robust with respect to systematic errors in ageing, alternative methods should be pursued especially for bluefin tuna. Population sizes of the older ages in the 1990 ICCAT assessment for bluefin tuna may be more positively biased as tracing back to the earlier years.

ACKNOWLEDGEMENTS

I would like to thank Z.Suzuki and N.Miyabe for their useful comments and suggestions.

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APPENDIX A: Derivation of equations (4) and (5).

Using Pope's approximation, $N_{t-1,y-1}+N_{t+,y-1}$ is given by

$$N_{t-1,y-1}+N_{t+,y-1}=N_{t+,y}\exp(M)+(C_{t-1,y-1}+C_{t+,y-1})\exp(M/2) \quad \text{(A1)}$$

Let $\alpha=1+\gamma$. Assuming $\gamma<1$, dividing equation (3) by equation (2) gives

$$\frac{C_{t+,y-1}}{C_{t-1,y-1}}=\frac{N_{t+,y-1}}{N_{t-1,y-1}}\alpha\frac{F+M}{\alpha F+M}\frac{1-\exp(-\alpha F-M)}{1-\exp(-F-M)}$$

$$\fallingdotseq\frac{N_{t+,y-1}}{N_{t-1,y-1}}\alpha\exp(-\gamma F/2) \quad \text{(A2)}$$

where the subscript on F is dropped for simplicity. The approximation in equation (A2) is similar to the following approximation used in Pope(1973).

$$\frac{F+M}{F}\frac{1-\exp(-F)}{1-\exp(-F-M)}\fallingdotseq\exp(M/2) \quad \text{(A3)}$$

Forethermore, equation (A2) is approximated by

$$\frac{C_{t+,y-1}}{C_{t-1,y-1}}\fallingdotseq\beta_{y-1}\frac{N_{t+,y-1}}{N_{t-1,y-1}} \quad \text{(A4)}$$

where $\beta_{y-1}=1+(1-F_{t-1,y-1}/2)\gamma$. From equations (A1) and (A4), we have

$$N_{t+,y-1}=\frac{C_{t+,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}N_{t+,y}\exp(M)$$

$$+C_{t+,y-1}\frac{C_{t+,y-1}+C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}\exp(M/2) \quad \text{(A5)}$$

$$N_{t-1,y-1}=\frac{\beta_{y-1}C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}N_{t+,y}\exp(M)$$

$$+\beta_{y-1}C_{t-1,y-1}\frac{C_{t+,y-1}+C_{t-1,y-1}}{C_{t+,y-1}+\beta_{y-1}C_{t-1,y-1}}\exp(M/2). \quad \text{(A6)}$$

APPENDIX B : Derivation of equation (10)

Relationships between N_{t+} and $\overline{N}_{t+}$ for recent three years are

$$N_{t+,y-1}=(1+\varepsilon)\overline{N}_{t+,y-1} \tag{A7}$$

$$N_{t+,y-2}=(1+\varepsilon)(1+\rho\varepsilon)\overline{N}_{t+,y-2} \tag{A8}$$

$$N_{t+,y-3}=(1+\varepsilon)\{1+\rho\varepsilon+\rho^2\varepsilon(1+\varepsilon)\}\overline{N}_{t+,y-3} \tag{A9}$$

where

$$\rho = 1-\frac{C_{t+}\exp(M/2)}{\overline{N}_{t+}}$$

$$=1-\frac{F}{F+M}(1-\exp(-F-M))\exp(M/2). \tag{A10}$$

From equations (A7) to (A9), we assume

$$N_{t+,y-j}=(1+\varepsilon)(1+f_{y-j})\overline{N}_{t+,y-j} \tag{A11}$$

where f_{y-j} is a function of ρ and ε. Substituting equation (A11) and

$$N_{t+,y-j+1}=(1+\varepsilon)(1+f_{y-j+1})\overline{N}_{t+,y-j+1} \tag{A12}$$

into equation (6) gives the recurrence formula for f

$$f_{y-j}=\rho\varepsilon+\rho(1+\varepsilon)f_{y-j+1} \tag{A13}$$

where $f_{y-1}=0$. Taking the difference of equation (A13), we obtain

$$f_{y-j}=\rho\varepsilon\frac{1-\{\rho(1+\varepsilon)\}^{j-1}}{1-\rho(1+\varepsilon)}. \tag{A14}$$

Table 1. Comparison between true N_{15+} and F_{15+} and estimates of N_{15+} and F_{15+} using incorect catch-at-age data. (data from Restrepo and Powers, 1991)

Year	N_{15+}		F_{15+}	
	True	Estimates	True	Estimates
1978	500	39641	0.39	0.059
79	554	33698	0.36	0.065
80	570	28459	0.47	0.101
81	535	23188	0.39	0.097
82	521	18949	0.46	0.134
83	478	14906	0.53	0.173
84	421	11264	0.48	0.180
85	383	8468	0.45	0.211
86	377	6186	0.54	0.344
87	316	3962	0.66	0.657
88	240	1859	0.70	2.306

Table 2. The values of $C_{10+}/(C_9+C_{10+})$ for bluefin tuna

Year	Ratio	Year	Ratio	Year	Ratio	Year	Ratio
1970	0.87451	1975	0.85064	1980	0.92342	1985	0.89540
71	0.84010	76	0.96485	81	0.80235	86	0.94144
72	0.95474	77	0.92594	82	0.79394	87	0.78774
73	0.88455	78	0.97456	83	0.85058	88	0.75204
74	0.92121	79	0.95953	84	0.82301	89	0.79289

Table 3. Estimates of stock size at age in 1990 (Bluefin tuna)

Age	N	Age	N	Age	N	Age	N
10	2760	16	648	22	150	28	192
11	2621	17	760	23	66	29	127
12	1686	18	579	24	271	30+	883
13	366	19	180	25	122		
14	617	20	88	26	164		
15	934	21	53	27	57	TOTAL	13324