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SOME RELATIONSHIP AMONG BIOLOGICAL REFERENCE POINTS IN GENERAL PRODUCTION MODELS

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Introduction

Biological reference levels of fishing intensity are important to determine management longterm objectives for the exploitation of a fish stock.

The most common reference fishing levels are designated by Fmax (Beverton and Holt, 1957) and Fo.1 (Gulland and Boerema, 1973).

The definition of those levels can be based on the absolute instantaneous rates of change of the longterm total yield, Y, with the fishing level, F, that is, the derivative dY/dF.

This rate decreases as F increases, being its highest value when F=0, i.e., when there is no exploitation.

To analyse the possible relationships among the different reference levels it is useful to express the absolute instantaneous rate, dY/dF, at the level F, as a fraction, r, of its highest value. Thus

$$r = \frac{(dY/dF)_F}{(dY/dF)_{F=0}} \qquad \text{.......(1)}$$

The non negative values of the fraction r vary from the value 1, when F=0, to the value zero, when F=Fmax (by definition of Fmax dY/dF=0).

The negative values of r correspond to values of F bigger than Fmax and, therefore, they have little interest for reference points for management purposes.

From the relation, Y=F.B, where B is the longterm annual average of the exploitable biomass by the fishing gears, one can derive the expression

$$dY/dF = B + F.dB/dF \qquad \text{.......(2)}$$

Note that $(dY/dF)_{F=0}$ = Bo, that is, the non-exploited biomass. Thus the level at which r=0.1 can be taken as definition of Fo.1.

The fraction r can be expressed also as

$$r = \frac{B}{Bo} + \frac{dB/dF}{Bo} . F \qquad \text{.......(3)}$$

This expression is useful to obtain the relations between biological reference points for the general production models used in fish stock longterm assessments.

The general production models

The general production models more commonly used in fish stock assessments are the Schaefer model (1954), the exponential or Fox model (1970) and the Pella and Tomlinson GENPROD model (1969).

Those models could be characterized by a linear relationship between a function of the biomass and the fishing level, or

$$f(B) = a - b.F$$

where a and b are constants, F is the fishing intensity level and f(B) is a function of the biomass, B, function to be defined for each type of model.

Alternatively one can characterize the models by the expression

$$B = B(a-b.F) \qquad \text{.......(4)}$$

where now the function B(a-b.F) is the inverse function of f(B).

The three types of models mentioned above will be treated separately.

SCHAEFER Model

The Schaefer model will be defined by the linear function

$$B = a - b.F \qquad \text{.......(5)}$$

To obtain the fraction r of the expression (3) one needs to calculate B/Bo and (dB/dF)/Bo.

It is easy to get from the basic expression (5) of this model,

Bo = a and dB/dF = -b

therefore

B/Bo = 1-(b/a).F and

$$\frac{dB/dF}{Bo} = -b/a$$

Substituting these values in equation (3) the fraction r will become

$r = 1 - (2b/a).F$ ....(6)

For F=Fmax ,r will be zero, and

$2b/a = 1/Fmax$

Thus the expression (6) can be written as

$r=1-F/Fmax$ or

$F/Fmax = 1 - r$ ....(7)

In the particular case of r=0.1, it will be F=Fo.1, and equation (7) gives

$Fo.1/Fmax = 0.90$

Relations between other biological characteristics at F=Fo.1 and F=Fmax can also be obtained.

For instance, the relation between biomasses at those fishing levels ,i.e., Bo.1/Bmax can be derived rewriting the basic relation of the model, equation (5), as

$B = a.(1 - (2b/a).F/2)$

and, as seen before 2b/a is aqual to 1/Fmax, thus B can be expressed as

$B = a.(1 - 1/2.(F/Fmax))$

For F=Fmax ,B will be Bmax and from the anterior expression results

$Bmax=a/2$

Dividing the two last expressions one gets

$B/Bmax=2-F/Fmax$

and from equation (7) it will be

$B/Bmax = 1 + r$ ....(8)

For the particular case of r=0.1, at which corresponds B=Bo.1, the expression (8) will give

$Bo.1/Bmax = 1.10$

The relation between Yo.1 and YMAX can be obtained multiplying Fo.1/Fmax by Bo.1/Bmax.

In general terms it would be

$Y/YMAX = (F/Fmax).(B/Bmax) = (1 - r)(1 + r)$........................(9)

In particular case of r=0.1 ,it will be

$Yo.1/YMAX = .99$

These results show that for the Schaefer model the relations Fo.1/Fmax, Bo.1/Bmax and Yo.1/YMAX are constants.

A pratical conclusion is that the value of Fo.1,as well as the values of the other characteristics at Fo.1, can be calculated from the values of Fmax ,Bmax and YMAX.

Note that adding equations (7) and (8) will be

$F/Fmax + B/Bmax = 2$ ....(10)

This relation shows that, in the Schaefer model, any change of the fishing level, F, relative to Fmax, will be "compensated" by the corresponding longterm change of the biomass,B. For example, a reduction of 20% of the fishing level,F, relative to Fmax, will cause an increase of 20% of the biomass,B, relative to the Bmax.

FOX Model

The basic equilibrium relationship of the Fox model can be written as

$B = e^{a - b.F}$ ....(11)

To obtain the fraction r given by the expression (3) one needs, as have been done for the case of the Schaefer model, to calculate B/Bo and

$\frac{db/dF}{Bo}$. From equation (11) it will be

$Bo=e^{a}$

$dB/dF= -b.B$

Therefore

$B/Bo = e^{-b.F}$

and

$$\frac{dB/dF}{Bo} = -b.e^{-b.F}$$

Substituting these values in the equation (3), the fraction r will become

$$r = e^{-b.F}.(1-b.F) \quad \text{.......... (12)}$$

For F=Fmax , r will be equal to zero, and

$$b = 1/Fmax$$

Thus the expression (12) can be written as

$$r = e^{-F/Fmax}.(1-F/Fmax) \quad \text{.......... (13)}$$

For a given value of r this equation can be solved for F/Fmax, by iterative methods, for instance.

To obtain Bo.1/Bmax one can start by substituting the value of b by 1/Fmax in equation (11) and get

$$B = e^{a - (F/Fmax)}$$

For F=Fmax this equation gives

$$Bmax = e^{a-1}$$

Thus

$$B/Bmax = e^{(1-F/Fmax)} \quad \text{.......... (14)}$$

This expression allows one to calculate Bo.1/Bmax once the value of Fo.1/Fmax was obtained.

Alternatively, one could see that both factors of the equation (13) can be expressed in terms of B/Bmax and get the equation

$$B/Bmax.\ln(B/Bmax) = e.r \quad \text{.......... (15)}$$

Given the value of r, one can solve this equation for the unknown, B/Bmax.

For the particular case of r=0.1 the relations are

Fo.1/Fmax = 0.781521
Bo.1/Bmax = 1.244183
Yo.1/YMAX = 0.972355

Also, Fo.1/Fmax + Bo.1/Bmax = 2.025704

As for the Schaefer model, the results show that the relations Fo.1/Fmax, Bo.1/Bmax and Yo.1/YMAX are constants.
However, a reduction in F relative to Fmax, in this model, is not equal to the corresponding increase in B relative to Bmax. For the particular case of r=0.1 ,the increase in the biomass ratio is 2.57% higher than the corresponding reduction in F ratio.

<u>GENPROD Model</u>

The basic equilibrium equation of the GENPROD model can be written as

$$B = (a - b.F)^{1/(m-1)} \quad \text{.......... (16)}$$

where m is a constant bigger than 1.
For m=2 the GENPROD is equivalent to the Schaefer model and FOX (197) has shown that the limit of the GENPROD model when m tends to 1 is the Fox model.

Following a procedure similar to the one used for the other two models, that is, calculating from equation (16) B/Bo and (db/dF)/Bo and substituting in equation (3) one obtains:

$$B/Bo=(1 - b/a.F)^{1/(m-1)}$$

$$\frac{dB/dF}{Bo} = - \frac{b/a}{(m-1)} (1-b/a.F)^{(2-m)/(m-1)}$$

and the expression of r

$$r = (1-b/a.F)^{(2-m)/(m-1)}.(1- \frac{b.\ m}{a.(m-1)} F) \quad \text{.......... (17)}$$

For F=Fmax (and r=0) this expression gives

$$b/a = \frac{m-1}{m} (1/Fmax)$$

By substitution in the expression (17), one gets finally

$$r = (1- \frac{m-1}{m} F/Fmax)^{(2-m)/(m-1)}.(1-F/Fmax) \quad \text{.......... (18)}$$

Given a value of r it is possible to calculate the solution, F/Fmax, of the equation (18) for any value adopted for the parameter m. Again iterative methods can be used to solve the equation.

The values of B/Bmax and of Y/YMAX could also be obtained as in the cases of the two models before.

As an example B/Bmax can be obtained from the equation

$$B/Bmax = (m - (m-1).F/Fmax)^{1/(m-1)} \quad \text{.......... (19)}$$

The table 1 presents a summary of the results.

m	Fo.1/Fmax	Bo.1/Bmax	Yo.1/YMAX	Bmax/Bo	Bo.1/Bo	Fo.1/Fmax+Bo.1/B:
1.0	0.781521	1.244182	0.972355	0.3678795	0.457709	2.025704
1.2	0.819995	1.193441	0.978616	0.4018776	0.479617	2.013436
1.4	0.848355	1.158613	0.982915	0.4312012	0.499595	2.006968
1.6	0.869888	1.133469	0.985991	0.4568778	0.517857	2.003357
1.8	0.886657	1.114599	0.988268	0.4796334	0.534599	2.001257
2.0	0.900000	1.100000	0.990000	0.5000001	0.550000	2.000000
2.2	0.910816	1.088420	0.991350	0.5183794	0.564215	1.999236
2.4	0.919724	1.079045	0.992424	0.5350822	0.577378	1.998770
2.6	0.927165	1.071323	0.993293	0.5503534	0.589606	1.998488
2.8	0.933457	1.064867	0.994008	0.5643895	0.601000	1.998324
3.0	0.938835	1.059401	0.994602	0.5773503	0.611646	1.998236
3.2	0.943476	1.054721	0.995104	0.5893678	0.621619	1.998197
3.4	0.947516	1.050674	0.995531	0.6005518	0.630984	1.998191
3.6	0.951059	1.047146	0.995898	0.6109948	0.639801	1.998205
3.8	0.954188	1.044045	0.996216	0.6207753	0.648118	1.998234
4.0	0.956969	1.041302	0.996494	0.6299605	0.655979	1.998271

Table 1 - Values of some biological characteristics for different values (For m=1, Fox model for m=2 Schaefer model)

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