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NOTE ON COHORT ANALYSIS AND AGE-SPECIFIC FISHING MORTALITY

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COHORT ANALYSIS

The most common way of estimating fishing mortality at each age is by using Cohort Analysis/Virtual Population Analysis. Essentially this consists of estimating the fishing mortality (F_i) at each age (i) of a year-class, given a knowledge of the fishing mortality (F_{i+1}) that the year-class sustained at age (i + 1). Given F_{i+1}, the population at the beginning of that year (N_{i+1}) may be calculated from the catch (C_{i+1}), since

$$N_{i+1} = C_{i+1} (F_{i+1} + M)/F_{i+1} \cdot (1 - \exp\{-(F_{i+1} + M)\}) \quad \ldots\ldots\ldots\ldots (1)$$

where M is the natural mortality. It follows that F_i can be estimated by solving

$$\frac{N_{i+1}}{C_i} = \frac{(F_i + M) \exp\{-(F_i + M)\}}{F_i (1 - \exp\{-(F_i + M)\})} \quad \ldots\ldots\ldots\ldots (2)$$

Hence, given the estimate of fishing mortality at the oldest age (t) of the year-class for which catch numbers are available, it is possible to estimate the fishing mortality at each of the younger ages. Errors caused by the wrong estimation of the fishing mortality at the oldest age cause errors in the estimates of fishing mortality; these are in the same direction as the original error but proportionally smaller. Such errors tend to approach zero asymtotically with decreasing age. If the formula for fishing mortality is used the other way around, so that from an estimate of fishing mortality at the youngest age that of older ages is estimated, the proportionate error will increase sharply with increasing age. Pope (1972) considers the former type of error and gives a formula for the proportionate error in estimates of fishing mortality and population size.

The important feature of this source of error is that the oldest age of each year-class is necessarily the most recent, and thus the fishing mortality currently acting on the population cannot be estimated by this technique. The method does, however, give fairly robust estimates of fishing mortality in previous years, which may then be regressed against estimates of fishing effort. The current year's fishing effort can then be used to estimate the current year's fishing mortality. The success of this technique will be related to the reliability of the fishing effort as an index of fishing mortality.

As a step towards management the evaluation of fishing mortality has two main purposes. One of these is to evaluate the likely values of the proportions of the fishing mortality for the fully recruited ages that act on the partially recruited ages of fish. These can be obtained from VPA without the use of fishing effort data. The values of these partial recruitment factors might, however, be distorted if the initial values of fishing mortality used in the VPA were seriously in error.

The other purpose of estimating fishing mortality at age is to obtain an accurate estimate of the current situation in the fishery relative to the Maximum Sustainable Yield situation and to estimate the current population of fish. An estimate of this parameter is a prerequisite of our being able to regulate a fishery by catch quota or by effort quota, and Pope and Garrod (1973) consider the effect of errors in such estimates on catch quotas and effort quotas. Essentially they show that the accurate estimation of fishing mortality is necessary in order to estimate a catch quota that will have the desired effect.

A possible alternative to VPA was suggested by Agger *et al.* (1971). This has been expanded by Pope (1974). The method consists of obtaining estimates of fully recruited fishing mortality (F_j) in each year (j) and modifying these to give estimates of fishing mortality at age (i) by partial recruitment factors (S_i) so that the fishing mortality (F_{ij}) for fish aged i in year j is given by

$$F_{ij} = S_j F_j \quad \ldots\ldots\ldots\ldots (3)$$

Using the relation between the catch (C_{ij}) of fish aged j in year i and fishing mortality, we obtain:

$$C_{ij} = \frac{S_i F_j}{S_i F_j + M} \cdot N_{ij} \,(1 - \exp\{-(S_i F_j + M)\}) \quad \ldots\ldots\ldots\ldots (4)$$

and this can be approximated as:

$$C_{ij} \simeq F_j S_i N_{ij} \exp\left\{ -\frac{(F_j S_i + M)}{2} + \frac{(F_j S_i + M)^2}{24} \right\} \quad \ldots\ldots\ldots\ldots (5)$$

Dividing C_{ij} by the equivalent of (5) for $C_{i+1,\, j+1}$ and taking natural logarithms, we obtain

$$\ln\left(\frac{C_{ij}}{C_{i+1,\, j+1}}\right) = \ln\left(\frac{F_j}{F_{j+1}}\right) + \ln\left(\frac{S_i}{S_{i+1}}\right) + \frac{F_j S_i}{2} + \frac{F_{j+1} S_{i+1}}{2} + M + \frac{(F_j S_i + M)^2}{24} - \frac{(F_{j+1} S_{i+1} + M)^2}{245} \quad \ldots\ldots\ldots\ldots (6)$$

Thus we obtain an estimate of $\ln\left(\frac{C_{ij}}{C_{i+1,\, j+1}}\right)$ based only on the unknowns F_j, F_{j+1}, S_i and S_{i+1}. For each combination of ages and years this estimate can be obtained and compared with the true value of $\ln\left(\frac{C_{ij}}{C_{i+1,\, j+1}}\right)$. The values of F_j and S_i can be modified until the sum of squares of the differences between the observed and the true values of these functions is minimized. This method is only suggested tentatively and it could be that solutions obtained by it are not unique. In any case, the estimates of fishing mortality obtained by this method for the most recent year would appear to have a coefficient of variation which is some multiple greater than 1 of the coefficient of variation of the catch at age data.

Table 1 shows the values of fishing mortality at each age which are obtained if the data of Fonteneau and Lenarz (1974) for the Eastern Atlantic fishery for yellowfin tuna are used and it is assumed that M = 0.8. The fishing mortality on the oldest ages seems unreasonably high but that on the younger ages agrees reasonably with the figures of Fonteneau and Lenarz. It could be that this might be explained by M increasing sharply with age. A further run of the model was made with M increasing in steps of 0.1 from 0.6 at age 1 to 1.1 at age 6, and this gave similar results for the younger ages and rather lower, but still very high estimates, for the 5 and 6 year-olds. These results should, however, be regarded merely as an illustration and be treated with extreme caution, because the number of years for which data are available is really not sufficient for the method.

The form in which the data are used in this method suggests a possible statistical test to see whether there are differences between fishing mortalities in different years and whether there are different partial recruitment factors at different ages. This is done by writing the matrix of $\ln\left(\frac{C_{ij}}{C_{i+1,\ j+1}}\right)$ for all ages and years. In the case of Fonteneau and Lenarz's yellowfin data, this is a 5 by 5 matrix which is shown in Table 3. If we make a two-way analysis of variance on these data, we find that there are very significant differences between the column totals and significant differences between the row totals. Consideration of equation (6) suggests that column totals are due mostly to partial recruitment effects and that row totals are due mostly to fishing mortality changes. The analysis of variance, therefore, possibly shows that there are significant differences between the partial recruitment factors of each age and between the fully recruited fishing mortality of each age/year. If this is the case, it is encouraging to be able to show that such differences as are found are not entirely due to 'noise' in the data.

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Table 1: Fishing mortality of yellowfin tuna in the East Atlantic as estimated by using the least squares method and assuming M = 0.8.

Age	Year 1967	1968	1969	1970	1971	1972
1	0.105	0.160	0.214	0.213	0.151	0.168
2	0.162	0.248	0.332	0.331	0.235	0.261
3	0.277	0.423	0.567	0.565	0.401	0.445
4	0.53	0.81	1.09	1.09	0.77	0.86
5	1.17	1.78	2.39	2.38	1.69	1.88
6	3.00	4.58	6.14	6.12	4.34	4.82

Table 2: Fishing mortality of yellowfin tuna in the East Atlantic as estimated by using the least squares method and assuming M = 0.6 for one-year-olds but that it increases by steps of 0.1 to 1.1 for six-year-olds.

Age	Years 1967	1968	1969	1970	1971	1972
1	0.108	0.1719	0.237	0.226	0.162	0.187
2	0.150	0.239	0.329	0.313	0.225	0.259
3	0.240	0.380	0.524	0.500	0.359	0.413
4	0.444	0.704	0.969	0.924	0.663	0.764
5	0.857	1.358	1.870	1.784	1.280	1.474
6	0.863	1.368	1.883	1.796	1.289	1.485

Table 3: Matrix of $\ln \frac{\text{Catch age i, year j}}{\text{Catch age i + 1, year j + 1}}$ and resulting anova

Years j \ Ages i	1	2	3	4	5	Totals
1	- 0.49	0.68	0.47	1.00	2.41*	4.07
2	0.63	- 0.13	0.49	0.98	2.41	4.38
3	0.89	0.75	1.14	1.28	2.88	6.94
4	0.93	0.51	0.79	1.72	3.41	7.36
5	0.39	0.76	0.51	0.98	2.02	4.66
Totals	2.35	2.57	3.40	5.96	13.13	27.41

* Assumed value

Anova table

Cause	DF	Sum sqs	Mean sqs	F Ratio	Level of Significance
Column effects	4	16.2689	4.0672	34.6410	< 1%
Row effects	4	1.9073	0.4768	4.0609	5%
Residuals	16	1.8786	0.1174		
Total	24	20.0548			