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NOTE ON THE MIXED SPECIES PROBLEMS

J. G. Pope
Min. of Agriculture, Fish & Food
Fish. Lab.
Lowestoft, Suffolk

NOTE: Figure 1 omitted due to its 3-dimensional nature.

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J. G. Pope

Ministry of Agriculture, Fisheries and Food
Fisheries Laboratory
Lowestoft, Suffolk

INTRODUCTION

Mixed species can create management problems, because where one species is caught as a result of a fishery directed at another species, it may not be possible to achieve the maximum yield for the directed fishery without going beyond the maximum sustainable yield (MSY) for the by-catch species. This note is concerned with a simple graphic solution to this problem.

This management problem has been recognized for some time. Ricker (1958) considered the problem for salmon stocks and these were again considered by Paulik et al. (1967). Suda (1972) considered mixed fisheries as a problem for tuna management. The problem was given more definition by Garrod (1973) in relation to mixed fisheries in the ICNAF area, and these ideas were developed by Brown et al. (1973) and Fukuda (1974). Basically, Garrod proposed a matrix of directed fisheries and by-catches which assumes that fishing effort directed at species A (f_i) produces a fishing mortality on species B (${}_iF_j$) such that:

$$ {}_iF_j = {}_iq_j \, f_i \, , $$

where the prefix of the catchability coefficient q refers to the directed fishery species and the suffix to the by-catch species. Practically, the species, at which the fishery was directed, was defined as the species which gave each month the highest catch for each nation/gear/vessel class recorded in the ICNAF statistics. The relative size of each ${}_iq_j$ was related to the size of ${}_jq_j$, the catchability of the species in the directed fishery for it. This enabled Brown et al. to consider the problem of maximizing the catch from the more southerly ICNAF areas, subject to the constraints of the catch quotas on each species and, where necessary, to some constraints as to the minimum catch to be permitted for some national catches of specific species. Fukuda studied the problem in more general terms and pointed out that the solution for any mixed fishery must lie in a cone defined by the by-catch vectors of each directed fishery. The approaches of Brown et al. and Fukuda were both concerned with getting the maximum yield in a year within the constraints of certain catch quotas, these being chosen in most cases to attain the MSY for each stock independently. Thus, the solution is not likely to be quite the same as that which would give the MSY for all stocks combined. The purpose of this note is to generalize the arguments of Brown et al. and Fukuda to the problem of attaining the MSY for all stocks combined. The arguments are mostly graphic but could fairly easily be solved formally.

METHOD

Let us consider three stocks of fish with parabolic yield functions. In the interests of topicality these can be named yellowfin, albacore and skipjack, but it must be stressed that these yield functions are purely imaginary. The relationship between yield and the total fishing mortality on each species (F) is:

$$ \text{Yellowfin yield} = A(2.8F_y - F_y^2) \qquad 0 < F_y < 2.8, $$

$$ \text{Albacore yield} = B(1.6F_a - F_a^2) \qquad 0 < F_a < 1.6, $$

$$ \text{Skipjack yield} = C(1.2F_s - F_s^2) \qquad 0 < F_s < 1.2. $$

In general A, B and C will be different and the total fishing mortality on each of the species, F_y (yellowfin), F_a (albacore) and F_s (skipjack), will produce an overall yield of:

$$ Y_t = A(2.8F_y - F_y^2) + B(1.6F_a - F_a^2) + C(1.2F_s - F_s^2) \, . $$

Thus surfaces of equal yield will form ellipsoids within the ranges given for F_y, F_a, F_s. However, by a suitable transformation of the values of F we could convert these into spheres and so, for simplicity, we can consider the case when A = B = C = 100. In this case the surfaces of equal yield will form concentric spheres within the range specified for F_y, F_a and F_s.

Table 1 shows the yield from each stock at various levels of fishing mortality. It is obvious that the overall MSY would occur if F_y could be made equal to 1.4, F_a equal to 0.8 and F_s equal to 0.6, and it would then equal 296 units.

Table 2 shows the by-catch rates of the various species in each directed fishery. It can be seen that there is a considerable mortality induced on skipjack as a result of fishing for yellowfin, and that fishing for albacore produces a substantial mortality on yellowfin. The by-catch rates for the other two species in the skipjack fishery are, however, comparatively low. The result of these by-catch rates is that, if we call the fishing mortalities directed at yellowfin, albacore and skipjack F_1, F_2 and F_3 respectively, we find that:

$$F_y = F_1 + 0.875\,F_2 + 0.400\,F_3 ,$$

$$F_a = 0.286\,F_1 + F_2 + 0.300\,F_3 ,$$

$$F_s = 0.714\,F_1 + 0.563\,F_2 + F_3 .$$

To attain $F_y = 1.4$, $F_a = 0.8$, $F_s = 0.6$, and hence the overall MSY, we need $F_1 = 1.026$, $F_2 = 0.6574$, $F_3 = -0.5025$. Clearly, this violates the practical constraint that $F_1, F_2, F_3 > 0$, and therefore the overall MSY cannot be attained in practice unless steps can be taken to change the by-catch rates, for example by closed areas.

Since the overall MSY cannot be attained, what is the maximum yield that can be attained by a mixture of fisheries directed at the three species? The three-dimensional model (Figure 1) is intended to clarify the solution of this problem, the two horizontal axes being the direction of increase of F_y and F_a respectively and the vertical axis indicating the direction of increase of F_s. The plane perpendicular to the line $F_y = 1.4$ passes through the overall MSY (which is unattainable). The contour lines indicate the yield for values of F_y, F_a, F_s which occur on the various plane surfaces. (NB Spheres intersect planes in circles.) These contour lines are parts of the surfaces of equal yield which are, of course, concentric spheres centred on the overall MSY. The line OA is the locus of possible values of F_y, F_a, F_s that would be created by a fishery directed at yellowfin. The equation of this line is (from Table 2):

$$F_A = 0.286\,F_y ; \qquad F_s = 0.714\,F_y .$$

This can be solved at any value of F_y.

Thus it can be calculated that the line OA cuts the $F_y = 1.4$ plane at $F_A = 0.4$, $F_s = 1.0$. Similarly the line OB is the locus of possible values of F_y, F_a, F_s that would be created by a fishery directed at albacore, and the equation of this line is:

$$F_y = 0.875\,F_a ; \qquad F_s = 0.563\,F_a .$$

It can be calculated that the line OB intersects the $F_y = 1.4$ plane at $F_a = 1.6$, $F_s = 0.9$. OC is the locus of possible values of F_y, F_a, F_s created by a fishery directed at skipjack, and it has an equation:

$$F_y = 0.4\,F_s ; \qquad F_a = 0.3\,F_s .$$

Clearly, the only values of F_y, F_a and F_s that can be attained by positive combinations of the three directed fisheries must lie within the three-sided pyramid with vertex at O and sides OA, OB, OC. Since the contour lines are continuous the maximum attainable yield must lie on one of the surfaces of this pyramid, and it can be seen from inspection that in this particular problem it lies in the sector OAB, which in the model is labelled "solution sector for highest attainable MSY". The contours indicate that this occurs at $F_y \simeq 1.25$, $F_a \simeq 0.80$ and $F_s \simeq 0.95$, and that the combined yield is about 283 units at this point. The precise parameters and combined yield can, of course, be calculated by finding the yield sphere to which the sector of the plane OAB is tangential.

DISCUSSION

A solution for the attainable MSY in this particular mixed fishery situation has been discovered. The situation is not particularly realistic, but it does indicate some of the more important points connected with this definition of a mixed fishery. The first point is that possible solutions lie within a pyramid whose sides are defined by the loci of species fishing mortality generated by directed fisheries. It can be seen that this pyramid might have more than three sides. If, for example, there were two directed skipjack fisheries (e.g. purse seine and long line) having different by-catch rates, the pyramid might have four sides or alternatively one of the four directed fisheries might be contained in the pyramid defined by the other three directed fisheries.

If a constraint were required to ensure some directed fishery on each stock, the pyramid would have its vertex at the levels of F_y, F_a and F_s on skipjack determined by this constraint. Thus, if a constraint was made that at least a fishing mortality of 0.3 should be generated in the directed fishery for skipjack, the pyramid would have its vertex at $F_y = 0.4 \times 0.3 = 0.12$, $F_a = 0.3 \times 0.3 = 0.09$, and $F_s = 0.3$. The attainable MSY must always lie on the surface of such pyramids, since the spheres of equal yield are continuous. This could, however, not necessarily be the case if a constraint on the maximum fishing mortality in any particular species were made, since the solution might then lie on the constraint plane in the interior of the pyramid. In another example the overall MSY might lie within the cone OABC, in which case, of course, the overall MSY would be attainable and the solution could be found by solving the simultaneous equations for the necessary levels of directed fishery.

The arguments given here can be extended to more than three stocks and the author hopes to present a more general mathematical argument in due course. It is hoped that this note and the model will have helped to clarify the situation. It is important to appreciate that the highest attainable MSY indicated by this method would only be attained by fairly complicated catch quotas on each species. It is interesting to see what the effect of a more simple regulation such as an overall effort quota might be. If for example the catchability of yellowfin in the directed fishery was such that a unit effort directed at it produced a fishing mortality $F_y = 0.0125$, and similarly if a unit effort directed at albacore produced an $F_a = 0.0140$ and a unit directed at skipjack produced an $F_s = 0.0130$, then an effort quota of 100 units of effort would, approximately, enable the maximum attainable yield to be taken. This can be seen by moving the flap labelled "Effort Quota Flap" (situated behind the plane $F_y = 0$) so that it touches the plane OAB at the maximum attainable MSY. It can be seen that the triangle formed by the intersections of this plane with the lines OA, OB, OC is the area of possible values of F_y, F_a and F_s if the effort quota was taken up precisely. It can be seen from the contours on this plane that this effort quota could give the attainable MSY of 283 units. This would be attained if the effort were directed selectively at only yellowfin and albacore. Alternatively, the yield might be as low as 150 if all the effort were directed at skipjack. In this case, incidentally, the skipjack stock would be destroyed. Although it is unlikely that effort would concentrate on one species to the exclusion of the others, it is possible that this might happen if one particular species were held in particular esteem by a large section of the fishing nations. This is in effect what is happening in world fisheries today, except that constraints are now being applied to the distribution of effort, and, indirectly, to its quantity.

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Table 1 Assumed yield functions for three tuna stocks

	Albacore	Yellowfin	Skipjack
F	100 $(1.6F-F^2)$	100 $(2.8F-F^2)$	100 $(1.2F-F^2)$
0	0	0	0
0.1	15	27	11
0.2	28	52	20
0.3	39	75	27
0.4	48	96	32
0.5	55	115	35
0.6	60	132	36
0.7	63	147	35
0.8	64	160	32
0.9	63	171	27
1.0	60	180	20
1.1	55	187	11
1.2	48	192	0
1.3	39	195	0
1.4	28	196	0
1.5	15	195	0
1.6	0	192	0
1.7	0	187	0
1.8	0	180	0
1.9	0	171	0
2.0	0	160	0
2.1	0	147	0
2.2	0	132	0
2.3	0	115	0
2.4	0	96	0
2.5	0	75	0
2.6	0	52	0
2.7	0	27	0
2.8	0	0	0

Table 2 Fishing mortality created on each species by a unit fishing mortality directed at each species

Unit fishing mortality directed at	Resulting units of mortalities on		
	Yellowfin	Albacore	Skipjack
Yellowfin	1.0	0.286	0.714
Albacore	0.875	1.0	0.563
Skipjack	0.400	0.300	1.0